Relative Computer Vision Based Navigation for Small Inspection Spacecraft

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Motivating Problem Statement: Diagnosing On-Orbit Failures

- On-orbit failures of large spacecraft can be challenging, time-consuming, costly and dangerous
- The ability to visually inspect the host spacecraft can be helpful in the debugging process

- Mission Concept of Operations [1]:
  - Small inspector satellite deployed from larger host spacecraft
  - Deployment is triggered by an on-orbit anomaly and commanded from the ground
  - Inspector satellite design should minimize requirements on host spacecraft
    - No shared power or communications
    - No navigational aids apart from a single small fiducial marker

- Problem statement:
  - How to design visual navigation system to estimate 12 DOF relative state?
    - Fiducial target must be as small and compact as possible (i.e. minimum number of points)
  - Accuracy and reliability are important for collision avoidance

Literature Review

• Spacecraft Visual Relative Navigation

• Augmented Reality
  – Kato and Billinghurst, “Marker Tracking and HMD Calibration for a Video-Based Augmented Reality Conferencing System,” IWAR, 1999
  – Abasa et. al. “Comparison between Particle Filter Approach and Kalman Filter-Based Technique for Head Tracking in Augmented Reality Systems,” ICRA, 2004

• Photogrammetry: Exterior Orientation and Absolute Orientation Problems
  – Horn, “Closed-Form Solution of Absolute Orientation using Unit Quaternions,” J. of the Optical Soc. of America, 1987
Approach and Contributions

• Overview:
  – Design of a 12DOF relative state estimator using a small fiducial target observed by a single monochrome camera

• Approach:
  – Space saving fiducial target design
    • Minimum number of coplanar points (four)
  – Solve the exterior orientation problem using an iterative globally convergent algorithm [3]
  – Relative pose is filtered with the dynamics using a Multiplicative Extended Kalman Filter (MEKF) [4]
    • Attitude Error Representation
    • Linear Measurement Model

• Primary Contributions
  1. Approach that works well for minimum number of point correspondences (four coplanar)
  2. Linearly fuses globally convergent exterior orientation solution with vehicle kinematics and dynamics
  3. MEKF formulation to estimate of angular velocity from orientation measurements

Target Detection: Contrasting Concentric Circles

- **Fiducial Target Design Requirements:**
  - Intended to be as simple, and small as possible
  - Image processing algorithms should require minimal computational complexity
  - Target detection and matching should be invariant to rotation and illumination conditions
- **Concentric Contrasting Circles (CCC):**
  - Based on research performed by [1,2] specifically for spacecraft environment
  - Inner and Outer “Blobs” have collocated centroids
  - Area ratio is invariant to rotation
- **Target Design:**
  - Four points with varying area ratios

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Steps for Target Detection:

1. Image Segmentation using Adaptive Thresholding
2. Connected Component Labeling
3. Filter Components by Area
4. Search for Collocated Centroids
5. Filter Components by Area Ratio and Color
6. Four Point Check
7. Area Ratio Sorting and Correspondences

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Globally Convergent Iterative Solution to Exterior Orientation Problem

- Exterior Orientation Problem (Resection Problem):
  - Solve for Translation and Rotation between a monocular camera frame and a known target object using point correspondences
  - Minimum number of planar point correspondences is 4
  - No exact, analytical solution is known

- Definitions, see diagram:

\[
\begin{align*}
x_n &= R_k y_n + T_k \\
v_n &= \begin{bmatrix} u_n^1 \\ u_n^2 \end{bmatrix}
\end{align*}
\]

- Minimize Mean Squared Re-projection Error:

\[
\epsilon_k^2 = \frac{1}{n} \sum_{n=1}^{N} \left| \left| R_k y_n + T_k - d_k^k v_n \right| \right|^2
\]

- 2 Step Iterative Algorithm [1]

  - Step 1: Assume pose is known, solve for base length (d):
    \[
    \{d^*_n\} = \arg \min_{d_n} \sum_{n=1}^{N} \left| \left| R_k y_n + T_k - d_n^k v_n \right| \right|^2
    \]

  - Step 2: Assume base length is known, solve for pose [2]:
    \[
    \{R_k^*, T_k^*\} = \arg \min_{R_k, T_k} \sum_{n=1}^{N} \left| \left| d_k^k v_n - (R_k y_n + T_k) \right| \right|^2
    \]

- Algorithm is shown to be globally convergent for any initial conditions [1]
- Requires >1000 iterations for random initial conditions,
  < 50 iterations if previous timestep’s solution is used
- Estimator is guaranteed to have a bias that decreases with every iteration

Continuous Time Kinematics and Dynamics

- Rigid Body Equations of Motion:
  - Process noise enters as forces and torques \( (W_v, W_\omega) \)
  - Implicit assumption that host spacecraft is an inertial frame (i.e. not accelerating)
  - Small angular velocity assumption

\[
\Omega(\omega) = \begin{bmatrix}
0 & \omega_3 & -\omega_2 & \omega_1 \\
-\omega_3 & 0 & \omega_1 & \omega_2 \\
\omega_2 & -\omega_1 & 0 & \omega_3 \\
-\omega_1 & -\omega_2 & -\omega_3 & 0
\end{bmatrix} \\
[\omega \times] = \begin{bmatrix}
0 & \omega_3 & -\omega_2 \\
-\omega_3 & 0 & \omega_1 \\
\omega_2 & -\omega_1 & 0
\end{bmatrix}
\]

\[
\dot{r} = v \\
\dot{v} = \frac{1}{m} W_v \\
\dot{q} = \frac{1}{2} \Omega(\omega) q \\
\dot{\omega} = J^{-1} ( -\omega \times J \omega + W_\omega ) \approx J^{-1} W_\omega
\]

- Multiplicative EKF uses attitude error to ensure Covariance matrix remains Positive Definite [1]:

\[
x = \begin{bmatrix} r & v & a_p & \omega \end{bmatrix}^T \\
a_p = \frac{4}{1 + q_4} \begin{bmatrix} q_1 \\
q_2 \\
q_3 \\
q_4
\end{bmatrix}^T = \frac{4}{1 + q_4} \bar{q}
\]

\[
\frac{da_p}{dt} = \dot{a}_p = \frac{4}{1 + q_4} \dot{\bar{q}} - \frac{4}{(1 + q_4)^2} q_4 \bar{q}
\]

\[
\dot{\bar{q}} = \frac{1}{2} \left( [\omega \times] a_p + \frac{4 q_4}{1 + q_4} \omega \right) + \omega \cdot \bar{q} a_p
\]

\[
\approx \frac{1}{2} [\omega \times] a_p + \omega
\]

Discrete Time Kinematics and Dynamics for Multiplicative EKF

- Continuous Linear state equations
  - Two related assumptions:
    - Low Angular velocity
    - Small change in angle per time-step
  \[
  \dot{x} = Ax + B_W W
  \]
  \[
  \begin{bmatrix}
  \dot{r} \\
  \dot{v} \\
  \dot{a}_p \\
  \dot{\omega}
  \end{bmatrix} =
  \begin{bmatrix}
  0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
  0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
  0_{3 \times 3} & 0_{3 \times 3} & \frac{1}{2}[\omega \times] & I_{3 \times 3} \\
  0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3}
  \end{bmatrix}
  \begin{bmatrix}
  r \\
  v \\
  a_p \\
  \omega
  \end{bmatrix} +
  \begin{bmatrix}
  0_{3 \times 3} & 0_{3 \times 3} \\
  \frac{1}{m}I_{3 \times 3} & 0_{3 \times 3} \\
  0_{3 \times 3} & 0_{3 \times 3} \\
  0_{3 \times 3} & J^{-1}
  \end{bmatrix}
  \begin{bmatrix}
  W_r \\
  W_v \\
  W_{\omega}
  \end{bmatrix}
  \]

- Discrete time equations
  - Standard discretization of kinematics and dynamics (details provided in the paper)
  - Integrals of matrix exponentials are solved numerically

- MEKF Reset Step applied after each iteration
  \[
  \hat{q}(k) = \delta q(\hat{a}_p^+(k)) \otimes q_{ref}(k) \quad \hat{a}_p^+(k) = 0_{3 \times 1} \quad q_{ref}(k + 1) = \hat{q}(k)
  \]

- Measurement Equation (Primary Contribution #2 & #3)
  - \( q_{ref}(k)^* \) is inverse of reference rotation
  - Measurements from Exterior Orientation: \( \tilde{r}(k), \tilde{q}(k) \)
  - Measurement Model is linear, unlike other approaches
  - Non-linear estimation is fully separated from MEKF by Exterior Orientation

  \[
  y(k) = \begin{bmatrix}
  \tilde{r}(k) \\
  \tilde{a}_p(k)
  \end{bmatrix} = H(k)x(k) + V =
  \begin{bmatrix}
  I_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
  0_{3 \times 3} & 0_{3 \times 3} & I_{3 \times 3} & 0_{3 \times 3}
  \end{bmatrix}
  \begin{bmatrix}
  r(k) \\
  v(k) \\
  a_p(k) \\
  \omega(k)
  \end{bmatrix} +
  \begin{bmatrix}
  V_r \\
  V_v \\
  V_{a_p}
  \end{bmatrix}
  \]

  \[
  \delta q(\tilde{a}_p(k)) = \tilde{q}(k) \otimes q_{ref}(k)^*
  \]
Fault Detection and Outlier Rejection

• The image processing algorithm may detect four markers that do not correspond to the actual target
  – May be due to background clutter in the visual image

• Measurement rejection method should incorporate accuracy of the current state estimate and the prediction based on the dynamic model

• Approach:
  – Reject based on the Mahalanobis distance, \( d(k) \), of the measurement innovation, \( i(k) \)

\[
    i(k) = y(k) - h(\hat{x}(k)) \\
    d(k) = \sqrt{i(k)^T (E[i(k)i(k)^T])^{-1} i(k)} \\
    T > (y(k) - h(\hat{x}(k)))^T (R + H(k)P(k)H(k)^T)^{-1} \times (y(k) - h(\hat{x}(k)))
\]

  – Using the linearized approximation to the innovation covariance, thresholds, \( T \), can be set for both the position and orientation measurements

  – This approach is similar to the gating concept frequently used in radar tracking systems [1].

Overview of Experimental Characterization of Accuracy and Precision

- Algorithm was implemented on the SPHERES satellites and Goggles and tested at the MIT Space Systems Laboratory glass table

- Multiple modes testing for error characterization:
  - Metric measurement
  - SPHERES Ultrasonic “Global Metrology” Navigation System [1]

- Full details available in [2]

- Experimental Results:
  - Video 1: Exterior Orientation Results
  - Test 1: Translation along Horizontal Plane
  - Test 2: Rotation of +/- 45 degrees about Vertical Axis
  - Test 3: Fault Detection and Outlier Rejection
  - Video 2: Multiplicative EKF Results

Exterior Orientation Video Results

Image Preprocessing: Adaptive Segmentation
Circle represents region of interest

Primary Contribution #1
- Experimental Validation of exterior orientation for 4 coplanar points

Raw Video with Overlay
Four X’s represent CCC Centroids
Box Outline represents Estimated Pose from Exterior Orientation
- Note: there is no Box Outline when X’s are lost
Estimate’s numerical values in bottom left
Translation along Horizontal Plane

Note: Y-axis is perpendicular to optical axis and in the horizontal plane (perpendicular to the gravity vector)
X-axis is parallel to optical axis (depth from camera)
Rotation of +/- 45 degrees about Vertical Axis

Note: Psi is a rotation about the Vertical Axis (parallel to the gravity vector)
Demonstration of Fault Detection and Outlier Rejection
Multiplicative EKF Video Results

Image Preprocessing: Adaptive Segmentation
Circle represents region of interest (Same as before)

Raw Video with Overlay
Four X’s represent CCC Centroids
Box Outline represents Estimated Pose from
Exterior Orientation + MEKF + Outlier Rejection
- Note: there is a Box Outline when X’s are lost (state propagation)
Estimate’s numerical values in bottom left
Summary of Upper Bounds on Accuracy and Precision

• Since the SPHERES Global Metrology system also contains estimation errors, only upper bounds on accuracy and precision can be established [1]

<table>
<thead>
<tr>
<th>State</th>
<th>Precision</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static Position</td>
<td>± 0.5 cm</td>
<td>± 1.5 cm</td>
</tr>
<tr>
<td>Dynamic Position</td>
<td>± 1.0 cm</td>
<td>± 2 cm</td>
</tr>
<tr>
<td>Static Linear Velocity</td>
<td>± 0.1 cm/s</td>
<td>± 0.1 cm/s</td>
</tr>
<tr>
<td>Dynamic Linear Velocity</td>
<td>± 1.0 cm/s</td>
<td>± 0.25 cm/s</td>
</tr>
<tr>
<td>Static Orientation</td>
<td>± 0.5 degrees</td>
<td>± 3.0 degrees</td>
</tr>
<tr>
<td>Dynamic Orientation</td>
<td>± 2.0 cm/s</td>
<td>± 5.0 cm/s</td>
</tr>
<tr>
<td>Static Angular Rate</td>
<td>± 0.5 degree/s</td>
<td>± 0.1 degree/s</td>
</tr>
<tr>
<td>Dynamic Angular Rate</td>
<td>± 5.0 degree/s</td>
<td>± 1.0 degree/s</td>
</tr>
</tbody>
</table>

Conclusions and Future Work

• Presented approach for single camera vision based navigation using a fiducial target including:
  – Fiducial Target Design
    • Invariant to rotation and lighting
  – Two estimator approach
    • Exterior Orientation
      – Globally convergent iterative algorithm for nonlinear pose estimation
    • Multiplicative EKF
      – Incorporates rigid body dynamics
  – Outlier detection and rejection
    • Mahalanobis distance incorporates uncertainty

• Key Contributions
  1. Approach that works well for minimum number of point correspondences (four coplanar)
  2. Linearly fuses globally convergent exterior orientation solution with vehicle kinematics and dynamics
  3. MEKF formulation to estimate angular velocity from orientation measurements

• Future Work
  – Improvements on Target Design
  – Incorporate additional information into MEKF
    • i.e. Control Forces and Torques
Questions

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• AIAA GNC Graduate Student Paper Competition
Perspective Projection & Collinearity Equation

\[
\begin{bmatrix}
p_x \\
p_y \\
p_z
\end{bmatrix} = \mathbf{R} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \mathbf{T}
\]

\[
z = h(\mathbf{R}, \mathbf{T}) = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} f \frac{p_x}{p_z} \\ f \frac{p_y}{p_z} \end{bmatrix}
\]

\[
\mathbf{H}(\mathbf{R}, \mathbf{T}) = \begin{bmatrix} \frac{\partial h(\mathbf{R}, \mathbf{T})}{\partial \mathbf{R}} & \frac{\partial h(\mathbf{R}, \mathbf{T})}{\partial \mathbf{T}} \end{bmatrix}
\]

\[
\mathbf{R}_{CG} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}
\]

\[
\mathbf{t}_{G/C} = \begin{bmatrix} x_{G/C} \\ y_{G/C} \\ z_{G/C} \end{bmatrix}
\]

\[
x_{p/C} = s f \frac{r_{11} x_{p/G} + r_{12} y_{p/G} + r_{13} z_{p/G} + x_{G/C}}{r_{31} x_{p/G} + r_{32} y_{p/G} + r_{33} z_{p/G} + z_{G/C}} + c_x
\]

\[
y_{p/C} = s f \frac{r_{21} x_{p/G} + r_{22} y_{p/G} + r_{23} z_{p/G} + y_{G/C}}{r_{31} x_{p/G} + r_{32} y_{p/G} + r_{33} z_{p/G} + z_{G/C}} + c_y
\]
Line of Sight Angles and Unit Vector:

\[
\begin{align*}
\theta_a &= \arctan\left( \frac{x'' - c_x}{sf} \right) = \arctan\left( \frac{x}{z} \right) \\
\theta_e &= \arctan\left( \frac{y'' - c_y}{sf} \right) = \arctan\left( \frac{y}{\sqrt{x'^2 + z^2}} \right)
\end{align*}
\]

\[
\begin{bmatrix}
 r_x \\
 r_y \\
 r_z
\end{bmatrix}
= \frac{1}{\sqrt{x'^2 + y'^2 + f^2}}
\begin{bmatrix}
 x \\
 y \\
 z
\end{bmatrix}
\]

\[
= \frac{1}{\sqrt{x'^2 + y'^2 + f^2}}
\begin{bmatrix}
 x'' \\
 y'' \\
 f
\end{bmatrix}
\]

\[
= \begin{bmatrix}
 \cos \theta_e \sin \theta_a \\
 \sin \theta_e \\
 \cos \theta_e \cos \theta_a
\end{bmatrix}
\]

Collinearity Equation:

\[
x_{p/C} = sf \left[ \frac{r_{11} x_{p/G} + r_{12} y_{p/G} + r_{13} z_{p/G} + x_{G/C}}{r_{31} x_{p/G} + r_{32} y_{p/G} + r_{33} z_{p/G} + z_{G/C}} + c_x \right] \\
y_{p/C} = sf \left[ \frac{r_{21} x_{p/G} + r_{22} y_{p/G} + r_{23} z_{p/G} + y_{G/C}}{r_{31} x_{p/G} + r_{32} y_{p/G} + r_{33} z_{p/G} + z_{G/C}} + c_y \right] \\
t_{G/C} = \begin{bmatrix}
 x_{G/C} \\
 y_{G/C} \\
 z_{G/C}
\end{bmatrix}
\]

Figure 3-4: Projection of a Point onto an Image Plane
# Absolute vs Exterior Orientation

<table>
<thead>
<tr>
<th></th>
<th><strong>Absolute Orientation</strong></th>
<th><strong>Exterior Orientation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solving for:</strong></td>
<td>Rotation (R) and Translation (T) between camera frame and target frame</td>
<td>Rotation (R) and Translation (T) between camera frame and target frame</td>
</tr>
<tr>
<td><strong>Target Points</strong></td>
<td>Known x,y,z coordinates</td>
<td>Known x,y,z coordinates</td>
</tr>
<tr>
<td><strong>Measurements</strong></td>
<td>Image coordinates (u,v) and <strong>depth (d)</strong> to target points</td>
<td>Image coordinates (u,v) to target point</td>
</tr>
<tr>
<td><strong>Typical Application</strong></td>
<td>Stereo Image of Known Target</td>
<td>Single Camera Image of Known Target</td>
</tr>
<tr>
<td><strong>Solution</strong></td>
<td>Exact Closed Form (i.e. Horn)</td>
<td>No Exact Closed Form Linear Methods and Iterative Methods</td>
</tr>
</tbody>
</table>
Discrete Time Process Model

\[ x(k) = \Phi(k)x(k - 1) + \Gamma(k)W(k) \]

\[ \Phi(k) = \begin{bmatrix}
I_{3\times3} & I_{3\times3} \Delta t & 0_{3\times3} & 0_{3\times3} \\
0_{3\times3} & I_{3\times3} & 0_{3\times3} & 0_{3\times3} \\
0_{3\times3} & 0_{3\times3} & e^{\frac{1}{2}[\omega\times] t} \Delta t & J^{-1}J - e^{\frac{1}{2}[\omega\times] \tau} d\tau \\
0_{3\times3} & 0_{3\times3} & 0_{3\times3} & I_{3\times3}
\end{bmatrix} \]

\[ \Gamma(k) = \begin{bmatrix}
\frac{1}{2m}I_{3\times3} t^2 & 0_{3\times3} \\
\frac{1}{m}I_{3\times3} \Delta t & 0_{3\times3} \\
0_{3\times3} & 0_{3\times3} \\
0_{3\times3} & J^{-1} \Delta t
\end{bmatrix} \]