Satellite Formation Flight and Realignment Maneuver Demonstration aboard the International Space Station

Christophe P. Mandy*, Alvar Saenz-Otero, David W. Miller
MIT Space Systems Laboratory, 70 Vassar St, Cambridge, MA, USA 02139

ABSTRACT
The Synchronized Position Hold Engage and Reorient Experimental Satellites (SPHERES), developed by the MIT Space Systems Laboratory, enable the maturation of control, estimation, and autonomy algorithms for distributed satellite systems, including the relative control of spacecraft required for satellite formation flight. Three free-flyer microsatellites are currently on board the International Space Station (ISS). By operating under crew supervision and by using replenishable consumables, SPHERES creates a risk-tolerant environment where new high-risk yet high-payoff algorithms can be demonstrated in a microgravity environment. Through multiple test sessions aboard the ISS, the SPHERES team has incrementally demonstrated the ability to perform formation flight maneuvers with two and three satellite formations.

The test sessions aboard the Space Station include evaluation of coordinated maneuvers which will be applicable to interferometric spacecraft formation missions. The satellites are deployed as a formation and required to rotate around a common center about a given axis, mimicking an interferometer. Various trajectories are then implemented to point the synthetic aperture in a different orientation by changing the common axis of revolution. Observation-time optimizing synchronization strategies and fuel balancing/fuel optimizing trajectories are discussed, compared and evaluated according to resulting mission duration and potential scientific output.

Keywords: SPHERES, Formation Flight, Space Interferometry, Optical Alignment

1. INTRODUCTION
As Astrophysics evolves, telescopes with increasingly greater resolution and collection power are needed for theories to be proved or induced from observations. Though space observatories have numerous advantages over ground instruments, their size is limited by launch vehicle constraints: a monolithic aperture must fit into the fairing, and mass has to be reduced as much as possible. The collection power of a space telescope can be improved by designing mirrors in modular pieces to be assembled in orbit, as it increases with aperture area.

Increasing angular resolution, on the other hand, can be done by flying multiple apertures in formation, and interfering the collected light. Several missions propose such schemes, such as JPL’s Terrestrial Planet Finder Interferometer (TPFI) or ESA’s Darwin. Light is collected from separated spacecraft distributed in a plane perpendicular to the line of sight to the object observed, then interfered in a combiner to form an image (Figure 1). Such a telescope's angular resolution is inversely proportional to its baseline, which in this case is simply the distance between apertures. Very small resolutions can be achieved by flying the satellites in wide formations, but this requires position accuracies inferior to the wavelength for the light to be combined. Operation of such a synthetic telescope aperture is different from the monolithic kind: when an image has been created for one target and a monolithic telescope must point at the next target, a canonical satellite slewing maneuver is performed with small thrusters or reaction wheels. For every new target to be observed by a distributed interferometer, the whole formation has to be relocated in a new plane perpendicular to the line of sight of the target and resized to a satisfactory baseline, balancing angular resolution with control authority to appropriately form an image. This simple task can be very costly in terms of propellant, which traditionally isn’t replenishable, and time, which increases ground operations cost and reduces the mission’s overall observation time.
2. SPHERES SATELLITES

The SPHERES (Synchronized Position, Hold, Engage and Reorient Experimental Satellites) testbed consists of six identical nano-satellites designed to demonstrate and mature control, estimation and formation flight algorithms in a fault-tolerant environment [2]. Three SPHERES are currently in the Space Station and operated by astronauts, while the other three are kept at MIT for ground testing. The satellites are actuated by 12 cold-gas thrusters operating with CO₂, compute algorithms on-line and are capable of intersatellite and satellite-to-laptop communication. Satellites have knowledge of their attitude and position from an infrared and ultrasound metrology system: US sensors around the satellite allow it to determine the time-lag between an IR ping and an US response from other satellites (relative metrology) or beacons attached to the Space Station frame (global metrology). In addition, each SPHERES contain 3 gyroscopes and 3 accelerometers.

Algorithms are first tested on MIT’s 3 degrees of freedom flat table facility where they are evolved to a space-testable level. Software is then sent to the Space Station to allow astronauts to test the designed experiments in the Space Station’s microgravity environment. Astronauts operate three SPHERES within a volume defined by five metrology beacons, communicating with the satellites via a laptop. In this manner, algorithms can be tested in an iterative manner, basing innovations on methods that have been tested. High-risk experiments can be attempted at low cost as any malfunction can be immediately tended to by the astronauts and is not mission-critical. Control and estimation algorithms and autonomy architectures can be run to the limits of their operability and tested for robustness without loss of precious hardware, while providing unique data on operation in a full 6 degrees of freedom environment. This procedure maximizes science time and limits overhead time by facilitating the modification of algorithms and their implementation and testing on hardware. Iteration of this process enables algorithms and formation-flight methods to be matured, optimized and improved for space applications.
3. OPTICAL REALIGNMENT TRAJECTORIES

For this section, we will assume that we start with a formation of \( n \) identical satellites of equal mass \( m \) symmetrically distributed equidistantly from a common center (i.e. the satellites form a regular \( n \)-gon), revolving around this centre at rate \( \omega_9 \). Assuming identical satellites is certainly reasonable as collectors in a formation serve the same purpose. The rotating maneuver of the formation follows the standard trajectory for Michelson interferometers ([3]), while symmetric distribution around a center corresponds to optimal configurations such as Golay or Cornwell arrays ([4]). Since these configurations have some sort of axial symmetry, most results pertaining to circular configurations are trivially generalizable by treating the Golary or Cornwell array as a set of smaller arrays arranged on concentric circles. We will also assume double-integrator dynamics for our spacecraft, i.e. allowing them to behave as they would in free space or, to first order, around one of the Lagrange points of a dynamical system. Trajectories for spacecraft subjected to gravitational dynamics can also be determined from the following discussions, but would not correspond to the environment on our testbed.

As for any telescope mission, the two primary performance metrics are its scientific output and operational cost. After a target has been observed for the amount of time required to generate an image, the whole array has to be reconfigured so that it lies in a plane perpendicular to the line of sight to a new target, and again maintained in formation revolving around its center to generate an image of the new target. No scientific data can be collected during the realignment and each realignment requires some satellites to use part of a limited propellant supply to reposition themselves. Our goal is to determine optimal trajectories for the satellites to follow as input for individual satellites’ tracking controllers.

We can also assume that the array does not need to be resized, with the formation changing from a circle in one plane to another circle in a new plane. Since our formation is not subjected to a gravitational field, these two circles will have identical centers to minimize fuel consumption. We will use this point as the origin of our coordinate frame, with the \( z = 0 \) plane defined as the plane of revolution of the array while observing the initial target, with the \( Z \) axis as axis of rotation. This treatment can be generalized to reconfigurations where the array needs to be resized by simply subjecting the trajectories to the required time-varying homothecy.

Finally, we will not take into account the required attitude control as the only condition is that the final attitude of the satellite allow light to be directed to the combiner. Attitude can be entirely decoupled from the trajectory optimization.

3.1 Baseline: Rigid Body Realignment

As a baseline, let us look at the trajectory the array would follow if it were a rigidly connected structure. The realignment maneuver consists of precessing the plane of revolution of the aperture at a rate \( \omega_p \), during which motion the satellites continue to rotate around their formation center and keep their relative distance. If we assume the plane is precessed around the \( X \) axis, then a satellite starting at \( \begin{bmatrix} x_0 \\ y_0 \\ 0 \end{bmatrix} \) will describe:

\[
\begin{bmatrix}
    x(t) \\
    y(t) \\
    z(t)
\end{bmatrix} = \begin{bmatrix}
    x_0 \cos(\omega_p t) - y_0 \sin(\omega_p t) \\
    x_0 \cos(\omega_p t) \sin(\omega_9 t) + y_0 \cos(\omega_9 t) \cos(\omega_p t) \\
    x_0 \sin(\omega_p t) \sin(\omega_9 t) + y_0 \sin(\omega_9 t) \cos(\omega_p t)
\end{bmatrix}
\]

(1)

which is simply a spherical helix (Figure 2).

An elementary optimization shows that the most fuel efficient precession axis is one which goes through as many nodes as possible: one \( n \) is odd, two if \( n \) is even.
Figure 4: Rigid Body optical 90° realignment trajectories for a 3-satellite array with $\omega_r / \omega_p = 2$ and unit radius. Initial and final array circles are dotted, with the initial formation denoted by the triangle.

### 3.2 Time-Optimal Realignment

Since the mission’s cost is an increasing function with time, and one goal is to maximize science output, an obvious optimization is to try to minimize the time spent for realignment maneuvers, so that the mission to be densely packed with observations. In this case, the cost function is simply the total time for the maneuver:

$$ J = t_f $$

with a constraint that the control command be bounded by the maximal thrust $u_{\text{max}}$:

$$ |u| \leq u_{\text{max}} $$

and constraints on final positions that the array be a revolving n-gon in the new plane by requiring that the final positions and velocities be in the new plane, positions belonging to a circle of the desired radius around the center with the velocities tangential to it, and that the center angle for two consecutive satellites be $\frac{2\pi}{n}$.

As might be expected, the optimal control is Bang-Bang, with satellites describing two parabolic arcs.

### 3.3 Fuel-Optimal Realignment

Optimizing for time may be detrimental to mission performance as it leads to a high cost in fuel. Once the fuel supply is depleted, the mission is over unless we assume on-orbit refueling capabilities which don’t exist yet. Mission duration can be extended by optimizing for fuel consumption. For electrical propulsion, the most likely candidate for on-orbit formation maintaining propulsion systems, the power required to operate a thruster is given by:

$$ P = \eta \frac{F^2}{2m} = \eta \frac{m^2 u^2}{2\dot{m}} $$

With $\dot{m}$ the propellant mass flow rate and $\eta$ an efficiency parameter. Fuel optimization then amounts to power optimization, leading to a quadratic cost function:
\[ J = \int_{t_0}^{t_f} u^2 dt \quad (8) \]

with \( t_f \) given for the problem to converge, or included in the cost function, to create a fuel-time optimization problem. An optimal controller is then of the Bang-Off-Bang form with satellites describing parabolic arcs and coasting for a period of time.

### 3.4 Fuel-Balanced Realignment

The purpose of minimizing fuel consumption was to extend mission lifetime. This does not effectively correspond to the actual metric for mission lifetime: if one satellite in the formation runs out of propellant, it is effectively useless and the mission could be compromised at that point, depending on the interferometer’s design. Maximizing mission lifetime then amounts to balancing fuel between the satellites. Following the approach in [5], and using the same fuel-energy equivalence argument as above, we can write the resulting cost function as

\[ J = \int_{t_0}^{t_f} \left( \sum_{j=1}^{n} u_j^T R u_j + \sum_{k=1}^{n} \sum_{l=1}^{n} (u_k - u_l)^T B (u_k - u_l) \right) dt = \int_{t_0}^{t_f} (u \bar{R} u) dt \quad (9) \]

with \( u_j \) the control command for the \( j \)-th satellite, \( u \) a vectorial concatenation of all command vectors, \( R \) and \( B \) the control and control difference weighting matrices and

\[
\bar{R} = \begin{bmatrix}
R + (n-1)B & -B & -B & \ldots & -B \\
-B & R + (n-1)B & -B & & -B \\
-B & -B & R + (n-1)B & & -B \\
& \vdots & & \ddots & & -B \\
-B & -B & -B & & R + (n-1)B
\end{bmatrix} \quad (10)
\]

subject to constraints on control command (5) and to final state constraints. Balancing fuel among the different satellites extends the total mission lifetime by allowing more observations to be performed. But in a realistic scenario, the mission’s lifetime is capped by operation costs, so that the goal is to pack the mission duration as densely as possible with actual observations rather than array pointing maneuvers. A full optimization problem then requires the \( R \) to \( B \) ratio to be determined first, so that maneuvering time is minimized (\( \bar{R} \)) with mission duration maximized (\( B \)), while taking into account various constraints resulting from multiple propellant sources feeding into different actuators, with some sources requiring higher weights: the light-gathering maneuvers during the interferometer’s operation generally use the same subset of actuators as the aperture is always pointing in the same direction. An analytical solution for this problem depends on the form of matrices \( R \) and \( B \), but a typical sample solution trajectory is shown in Figure 5.

### 4. RESULTS

Demonstrating a multiple satellite formation realignment maneuver requires a 6DOF testbed, such as the SPHERES satellites as the trajectories are inherently 3-dimensional. Realignment tests were carried out on the International Space Station with three-satellite formations by US astronaut Sunita Williams on March 24th and April 27th 2007. Tests were composed of 6 maneuvers:

1. A brief estimator convergence period for the SPHERES positioning system
2. A mutual identification maneuver and motion towards starting positions determined online an equilateral triangle of 0.8m side, in the plane in which the SPHERES are initially released
3. A synchronization maneuver during which the SPHERES consensually determine a starting time for the formation

![Fuel-Balanced 3-satellite optical realignment](image)

Figure 5: Fuel-Balanced optical 90° realignment trajectories for a 3-satellite array with $R = I$ and $B = 0.5*I$, $\omega_I = 1$ and unit radius. Initial and final array circles are dotted, with the initial formation denoted by the triangle.

4. A portion of a circular maneuver in the initial plane while always pointing the same face towards the center of the formation

5. The plane change maneuver to a plane rotated 90° from the initial plane while always pointing the same face towards the center of the formation

6. Another half-circle in the target plane while always pointing the same face towards the center of the formation

Only metrology for the final three maneuvers are shown in Figures 6 to 11.

![Estimated States](image)  ![Target States](image)

Figure 6: Rigid Body realignment of 3 SPHERES satellites. Estimated State (left) and planned trajectory (right).

Figure 6 shows the planned trajectory and actual trajectory of the three satellites for a rigid-body-like plane change maneuver. The satellites perform a 0.4m radius semicircle starting at 60 seconds, then after 90 seconds, change to a new
plane rotated 90° from the initial plane, such that the precession axis passes through one of the satellites. $\omega_x / \omega_y = 2$ and after another 90 seconds, the SPHERES execute a second semi-circle in the new plane. States and targets are determined in the global coordinate frame, fixed with regard to the Space Station, and measured with the global metrology system.

As Figure 7 shows, the SPHERES stay within 7cm of the desired trajectory at all times. The first increase in error, at 60 seconds, is due to the change from a static position in the rotation plane to a moving trajectory. The error increases again right at the start of the plane change maneuver (150s) and damps out near the end of it (270s), as the satellites have less actuation to perform at that stage. The controller used is a simple PID controller tracking the trajectory from Figure 6 recalculating its next target for each control period, during the semicircular maneuvers and for the final 70 seconds of the plane change maneuver. The first 20 seconds of the realignment maneuver use a PD controller as the targets tend to be farther from the satellites. There is a slight resonance between the errors of the satellites after the end of the rotation, as can be seen from the relative position plot, during which the PID controller causes the satellites to oscillate around their targets at the same frequency. A few gaps in the trajectories are indications of some communication packets lost during transmission of the satellite’s telemetry.

Figure 7: Position errors and relative distances between SPHERES satellites for a rigid-body realignment maneuver

Figure 8 superimposes the executed and desired trajectory for a single SPHERE.

Figures 9 to 11 correspond to a fuel efficient plane change. In this case, the spheres execute a full circle. The satellites perform a half-circle for 90 seconds in an initial plane, follow a fuel-efficient trajectory to a plane rotated 90° from the
first plane then execute five-sixths of a circle in the new plane for a period of 150 seconds. The plane change time was set to 30 seconds, and the corresponding trajectory is a pair of parabolic arcs separated by a coasting phase. During the test, one of the satellites ran out of batteries, so only the telemetry for two satellites is plotted, though the trajectories were determined for a 3-satellite array. Figure 9 shows the trajectories followed by the two satellites. In both cases, the coasting phase for all three axes intersects for 6 seconds, after which the control is inverted.

Figure 8: Executed (continuous) and Desired (dotted) trajectory for a rigid body realignment, showing very small errors

Figure 9: Fuel efficient realignment of 2 SPHERES in a 3 satellite formation

Figure 11 shows the error and relative distance of the two satellites over the course of the test. The error increases after 150 seconds, when the plane change maneuver is initiated. The trajectory requires maximum thrust at this point, but some small control lags cause the SPHERES not to reach their first few targets on time.

5. CONCLUSIONS

We have shown an elementary way to generate optimal trajectories for satellites in distributed interferometers to follow, during aperture realignment phases. These were successfully implemented using the SPHERES facility on board the Space Station, in a 6 degrees of freedom environment Future work includes generating trajectories for cost functions which combine multiple requirements such as fuel-balancing and time-optimizing, including additional constraints on velocities or distances to prevent plume impingement, constrained attitude control (such optimizing fuel consumption while forcing individual satellites to avoid pointing in the direction of the sun), expanding the fuel balancing approach to
cases where the satellites don’t start with the same amount of propellant and optimizing trajectories for discrete controllers or nonlinear control schemes. Another interesting investigation concerns retargeting apertures flying in a non-circular formation: trajectories generated by subdividing the formation in concentric circles may no longer be optimal as swapping the positions of certain apertures in the circles may lead to better solutions or more fuel-balanced formations.

![Graphs showing position error and relative distance between SPHERES for a 3-satellite fuel-efficient realignment test.]

Figure 11: Error and relative distance of two SPHERES for a 3-satellite fuel-efficient realignment test

REFERENCES