Design and Shape Control of Lightweight Mirrors for Dynamic Performance and Athermalization

by

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Submitted to the Department of Aeronautics and Astronautics in partial fulfillment of the requirements for the degree of

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Abstract

The next generation of space telescopes will need to meet increasingly challenging science goals. For these new systems to meet resolution goals, the collecting area of the primary mirror will need to be increased. However, current space telescope designs are reaching their limits in terms of size and mass. Therefore, new systems will need to include technologies such as lightweight mirrors, segmented or sparse apertures and active optical control. Many of these technologies have no flight heritage, so determining what combinations of technologies will create favorable designs requires detailed modeling and analysis. This thesis examines the design of a lightweight mirror for an advanced space telescope for both dynamic performance and shape control.

A parametric model of a rib-stiffened mirror is created in order to quickly analyze many different mirror geometries. This model is used to examine the homogeneous dynamics of the mirror to determine what geometry will maximize the ratio of stiffness to areal density. The mirror model is then used in a full dynamic disturbance-to-performance analysis so that system performance can be examined as a function of changes in the mirror geometry. Next, a quasi-static shape control algorithm is developed to control the mirror using the presence of thermal disturbances. The traditional method of mirror shape control relies on feedback from a wavefront sensor in the optical path. A wavefront sensor reduces the amount of light available for image formation, which causes problems when viewing very dim objects. Therefore, this control algorithm uses feedback from sensors embedded in the primary mirror. Control algorithms using both strain gages and temperature sensors are developed and compared to determine which sensor type results in better performance. The shape control algorithm with temperature sensors is analyzed using the parametric rib-stiffened mirror model to determine what geometries are best for shape control. The dynamic analysis is combined with the thermal control analysis in order to determine what mirror geometries will be favorable for both of these problems.

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Chapter 1

Introduction

The science goals of future telescope missions are becoming more and more ambitious. The James Webb Space Telescope (JWST), shown in Figure 1-1(a), will look back to the origins of the universe to study the formation of the first stars. The Terrestrial Planet Finder, shown in Figure 1-1(b), will search for Earth-like planets orbiting nearby stars. To achieve these science goals, telescopes will need better resolution, which will require new technologies and major design changes. The NASA advanced telescope and observatory roadmap points to some of the new architectures and key technologies which will be necessary to meet the science goals [10]. These technologies include lightweight, segmented or sparse apertures and active optical control.

In order to see these distant stars and Earth-like planets, the collecting area of the primary mirror must be increased [5]. However, as the size of the primary mirror is increased, the size of the launch fairing becomes a constraint. Both segmented and sparse apertures are potential technology options for increasing the mirror diameter. The James Webb Space Telescope, will have a primary mirror with a diameter of 6.5 meters, six times the collecting area of the Hubble Space Telescope [27]. Therefore, the JWST primary mirror is formed from 18 separate segments. This type of segmented system must then be deployed on orbit and will require active controls to ensure that each segment is aligned correctly. Missions such as the Terrestrial Planet Finder (TPF), will use a formation flown interferometer to work as a giant virtual telescope.

As the primary mirror becomes larger, it becomes important to design lightweight
mirrors. The primary mirror is usually the largest and heaviest component of the telescope, so reducing its mass will have a significant impact on the entire system. By using advanced materials and design techniques, the areal density of the primary mirror on JWST will be more than nine times smaller than the areal density of the Hubble primary mirror. The increased diameter and reduced mass of these new mirrors cause them to be more flexible than traditional mirrors. As a result, pre-launch validation becomes difficult for these large, flexible mirrors. It is impossible to completely replicate a space environment on Earth. The gravity field on Earth has a major effect on the flexible structures being designed. In addition, the next space telescopes will be approaching size limitations of environmental testing chambers. As system validation prior to launch becomes more difficult, the ability to correct defects on-orbit becomes more important.

Active control of the primary can be used on-orbit to correct for defects in the mirror. A manned servicing mission was used to correct the spherical aberration in the Hubble Space Telescope. It is unlikely that servicing missions will be possible for future space telescopes, making the ability to remotely perform corrections on-orbit more important. In addition, deployable mirrors, such as those being designed for JWST, will require initial alignment after launch and will use active control to maintain position. The primary mirror of a space telescope could have errors due to manufacturing, launch, deployment, spacecraft dynamics and changes in the thermal environment. Actuators embedded in the primary mirror can be used to correct the
mirror shape on-orbit.

This thesis will study the design of a lightweight primary mirror with embedded active controls. A parametric finite element model of the lightweight mirror has been created so that various geometric parameters can be changed easily. Using this model, the dynamic performance of the mirror will be studied as the geometric parameters of the mirror are changed. Following this, active controls will be applied in order to control the mirror for thermal disturbances using only sensors and actuators on the primary mirror.

1.1 Motivation

In order to meet the tightened science requirements, future space telescopes will utilize many advanced technologies. Design possibilities now range from the traditional very stiff telescope design to extremely lightweight, highly actuated systems. Determining the best combinations of these new technologies will be a challenge. In the traditional design process, as shown in Figure 1-2, a point design is chosen during the conceptual design phase. This decision is often based on expert knowledge and simple models based on previous missions. Detailed models are usually not created and analyzed until the preliminary design phase. Discovering that a design cannot meet mission requirements during the design phase can lead to costly redesigns, or even a return to the conceptual design phase.

![Figure 1-2: Conceptual and Preliminary Design Phases](image)

The next generation of space telescopes will be radically different from previous designs. Many of the new technologies which will be utilized have no flight heritage,
and so choosing a point design based on expert knowledge can be very difficult. Even if a design is chosen which meets the requirements, there is no guarantee that it is an optimal design. By modeling many architectures early in the design process, favorable families of designs can be identified. In order to explore the large space of potential architectures during the conceptual design phase, it is necessary to quickly build and analyze many models. A parametric model can be used to build and analyze models of systems which include different technologies. This thesis describes the Modular Optical Space Telescope (MOST) tool which is used to create a structural model of a space telescope based on parametric inputs. Analysis is then performed on the structural models so that many different architectures can be compared. The modularity of the parametric model allows new technologies to be included if they are interesting to the user. This parametric model will be used to evaluate the effects of changing primary mirror geometries. This approach is meant to augment rather than replace the role of expert opinion.

The primary mirror design is a major area of technology development for space telescopes. Major advances have been made recently in lightweighting mirrors using both new materials and different designs [17],[20]. There are many potential design options for creating a lightweight mirror. By using a parametric model, it is possible to determine what the optimal geometry of the mirror should be to maximize the stiffness to mass ratio of a primary mirror. The parametric model can also be used to determine how these changes in geometry will affect the overall performance of the system.

There are many potential sources of errors for the primary mirror, including optical manufacturing errors, deployment errors, defects due to launch, spacecraft induced vibrations and thermal deformations. In this thesis, control for quasi-static thermal disturbances will be discussed. These thermal disturbances could be caused by heat radiating from the Earth, or from errors in the heaters used to control the thermal environment. This mirror is designed with embedded actuators in order to directly correct for errors.

In general, a wavefront sensor, such as a Shack-Hartmann sensor, is used to control
the mirror shape [15]. Most wavefront sensors divide the available light between the
imaging detection and the wavefront measurement system using a beam splitter. In
order to operate, a wavefront sensor must have sufficient light to overcome background
noise, and still have enough light left over to form an image. Many scientifically
interesting stars are not bright, and cannot be seen once the light is split. If the
wavefront error could be measured by using sensors embedded in the primary mirror,
all of the available light could be used for the image. In addition, embedded sensors
remove the potential for errors to be introduced between the mirror and the sensor.

1.2 Previous Work

An overview of the literature shows that many aspects of space telescope design have
been the focus of research in the past decade. Parametric modeling of structures has
been used in a number of fields, including the design of telescopes, in order to improve
the design process. There have also been many advances in the design of subsystems
for space telescopes. Due to the technology advances which will be necessary for
future missions such as JWST, the design of lightweight mirrors has been a focus. As
lightweight mirrors are designed, the need for active shape control of these mirrors
has become increasingly important.

1.2.1 Parametric Modeling

Parametric modeling has been used in a number of fields to improve the design of
structures. In the area of automotive design, Botkin [7] uses an MSC Nastran opti-
mization routine (SOL 200) to study different realizations of the front structure of an
automobile. The goal of this optimization is to reduce the weight of the structure.
Feynes [11] develops a high level model of an entire automobile, and uses a hierarchi-
cal parametric CAD model to optimize the vehicle structure. This model also allows
for the easy addition of new features.

Parametric modeling has also been used in the field of aircraft design. Puorger [24]
has developed a code for the conceptual design of aircraft configurations. This code,
MAGIC (Multidisciplinary Aircraft desiGn of Innovative Configurations), includes structural analysis using FEA, buckling analysis, an aerodynamics model and an aeroelasticity model. Smith [28] automatically generates a model of an aircraft based on parametric inputs describing the geometry. This model is then analyzed using CFD. Baker [4] discusses the importance of parametric modeling early in the design cycle of aerospace vehicles. The parametric models are used to improve the weight and performance estimation and identify major problems before the detailed design phase is reached.

Recently, integrated models have been developed for both ground-based and space telescopes. Angeli et al [1], [2] created an integrated model for a ground-based telescope. This model includes an integrated structures-optics-controls model, a parametric cost estimation model and a science merit function. Miller, de Weck and Mosier [23] present the DOCS (Disturbance-Optics-Controls-Structures) framework for integrated modeling, simulation and analysis of optical telescopes, either ground-based or space. This framework is shown to be an efficient method for evaluating various missions. Uebelhart [32] [33] has expanded on this early DOCS framework to create a fully parameterized model for space telescopes. This work also includes an uncertainty model which can be used to identify critical parameters.

### 1.2.2 Lightweight Mirror Design

An overview of the literature shows that the design of lightweight mirrors is a major focus of research. Large reductions in the areal density of mirrors are required for the larger apertures which will be used in future space telescopes. Baiocchi and Burge [3] have been working at the University of Arizona to optimize the design of lightweight, active mirrors subject to thermal disturbances. They developed a relationship between key design parameters for a lightweight mirror (such as facesheet thickness, number of support points and number of actuators) in order to determine the best geometry for surface quality given a target mass. There has also been research into new material options for lightweight mirrors. Matson and Mollenhauer [22] state that monolithic glass mirrors are reaching their limits in terms of areal
density. They discuss some of the advanced materials which can be used to improve space telescope mirrors, including composite materials, foams and microsphere arrays. These materials will require new manufacturing and polishing techniques to reach the required surface quality.

Some lightweight mirror designs are currently being built and tested in order to demonstrate enabling technologies for the JWST. The Subscale Beryllium Mirror Demonstrator (SBMD) is a 0.532-meter lightweight mirror developed to show that surface quality and areal density goals for JWST could be met using a beryllium mirror [25]. It was tested at cryogenic temperatures so that the changes in surface quality could be accounted for as it undergoes cooling. A mirror areal density of under 10 kg/m² was achieved for this mirror, not including the actuators or support structure. The Advanced Mirror Systems Demonstrator (AMSD) program extended the work of the SBMD by testing a 1.4-meter hexagon similar to the segments which will be used for the JWST mirror [20]. This mirror will also include cryogenic actuators and a lightweighted support structure.

1.2.3 Mirror Shape Control

Deformable mirrors were first used in adaptive optics systems for ground-based telescopes. The capability of ground based telescopes is limited by the atmospheric turbulence which causes stars to twinkle [31]. The performance of these telescopes can be improved by the addition of a deformable tertiary mirror which corrects the errors in the wavefront. A traditional adaptive optics system includes a deformable mirror, a wavefront sensor and a control system. Many different designs exist for the deformable mirror. Freeman and Pearson [13] provide a review of several options for the deformable mirror design for different applications. Segmented mirrors, with piston, tip and tilt control of each segment, are one way of creating an active mirror. It is also possible to design a continuous surface mirror with either surface-normal or surface-parallel actuation. Surface-normal actuators must be mounted to a massive backstructure in order to function, making the deformable mirror too heavy for a space application. Therefore, the shape control work in this thesis utilizes surface-
parallel actuators.

Although ground-based telescopes utilize adaptive optics systems to correct for turbulence in the atmosphere, the current large telescopes being designed are also subject to low spatial-frequency disturbances such as gravity sag, thermal changes and wind-buffeting. To correct for these disturbances, a separate active optics system can be used. Angeli [1] describes a linear optical model developed for the 30-meter class of ground based telescopes. This model, which describes optical path differences in terms of primary and secondary mirror displacements, is a convenient way to design and analyze active control systems.

Space telescopes are able to perform significantly better than ground-based telescopes because they are not limited by atmospheric effects or gravity. However, there are still errors in the primary mirror due to fabrication errors, launch and deployment dynamics, thermal changes and on-orbit spacecraft dynamics. In order to reach diffraction-limited performance, on-orbit shape control of the primary mirror has become an important part of space telescope design. Robertson [26] initially developed the active optics concept for space telescopes. He describes the fundamental concept of an active optics system for a space telescope. A thin, deformable mirror was then developed in order to analyze and experimentally test the active optics concept.

Many different studies have been performed to show that the shape of a primary mirror can be controlled in order to improve the imaging capabilities of a space telescope system. In general, a control algorithm is developed based on the influence function of each actuator and the surface correction in the mirror is measured for some form of disturbance. Kapania et al [19] showed that a thin hexagonal segment could be controlled for thermal deformations using either force actuators or piezoelectric strips bonded to the back surface of the mirror. The mirror showed smaller deformations due to the thermal disturbances when it was mounted on force actuators. However, the force actuated system was much heavier than the system actuated by piezoelectrics and is probably not useful for a space application.

Furber et al [14] have worked on the problem of shape control for a one meter deformable mirror. They created a finite element model of a deformable mirror using
Nastran. This model was then used to determine the influence of each actuator in the system. The model developed was then used to simulate the performance of an active control system for various aberrations. The performance of the control algorithm is then analyzed as a function of the number of actuators, the actuator influence functions and the disturbance. Doyle et al [9] present an integrated model which is used to study the active control of a primary mirror. Design trades are performed by coupling the structural and optical design tools. To achieve this, a commercial code, SigFit is used. This code combines the disturbance and structures with a CODE V optics model in order to determine the best surface correction for a system of apertures.

1.3 Thesis Objectives

This thesis aims to address some issues which remain open in the area of lightweight mirror design for space telescope applications. Parametric modeling techniques will be applied to the design of a rib-stiffened mirror in order to determine favorable geometries for both dynamic performance and control of thermal effects. A particular goal of this model will be to use the desired mirror areal density as an input to the parametric model. This will allow mirrors of equal mass to be compared.

While active control of primary mirrors has been studied for many years, previous control systems have used wavefront sensors located elsewhere in the optical path to measure errors in the mirror. As mentioned earlier, these wavefront sensors usually split the light. In addition other errors could be introduced between the mirror and sensor since they are not collocated. To address this issue, this thesis will present an algorithm for shape control using only sensors embedded in the mirror.

1.4 Thesis Roadmap

This thesis will discuss the design of a lightweight mirror for both dynamic performance and athermalization. Chapter 2 describes the development of a parametric
model for a lightweight mirror. This mirror is part of a larger integrated model which includes structures, controls and disturbances. The parametric mirror model is used to study the homogeneous dynamics of the mirror as the geometric properties of the mirror change. Following this, a disturbance to performance analysis of the entire space telescope is performed to determine the effect of the mirror geometry on the telescope performance.

Chapters 3 and 4 then describe the development of control algorithms which can be used to minimize the effects of thermal disturbances on a lightweight mirror using only embedded sensors and actuators. In Chapter 5 these algorithms are analyzed on a circular flat plate with surface-parallel actuators, which is representative of a lightweight mirror. The shape control is analyzed with a varying number of actuators and for a number of potential thermal disturbances. In Chapter 6, the shape control algorithm presented in Chapter 4 is used to control the full rib-stiffened mirror which was presented in Chapter 2. Again, the results of this control are analyzed for different disturbances. The parametric mirror model is then used to alter the geometric properties of the mirror in order to determine favorable geometries for shape control.
Chapter 2

Lightweight Mirror Optimization for Dynamic Performance

Advanced space telescopes concepts include increasingly tight requirements on optical performance. In order to create high resolution systems, large apertures are necessary. However, current designs are reaching their limits in terms of mirror diameter and mass. Primary mirror diameter is constrained by the size of current launch fairings. The overall system mass is a major driver of launch costs. If these new telescope designs are to succeed, advanced technologies, such as deployable optics and low areal density mirrors, must be incorporated.

Recently, many new technologies have been developed which can be used to improve the design of the primary optics. In particular, segmented mirrors, lightweighting and active controls can be used to meet performance requirements while minimizing mass. It is necessary to use segmented mirror systems to achieve large diameters; this way mirrors can be folded into a launch configuration and then deployed on orbit. As the primary mirror diameter increases, the mass of the system will become large. Lightweighting techniques have been developed in order to reduce this mass; however, this can cause the mirror to become flexible. Active controls are then added to the mirror so that the optical performance requirements are met. Because these are new technologies, understanding the appropriate combinations of these technologies requires detailed modeling and analysis.
The Modular Optical Space Telescope (MOST) project has been working to develop a parametric model of a space telescope. This includes structural model generation, controls models and a dynamic disturbance to performance analysis. Using this tool, features of the telescope can be easily changed in order to determine the effects of particular parameters on performance metrics. In particular, different mirror geometries can be analyzed to find those that provide the highest stiffness for a given areal density. In this chapter an overview of the MOST model will be provided for background. Then, the structural modeling and auto-meshing of the lightweight mirror will be discussed in detail. Finally, the dynamic performance of the lightweight mirror, both attached to the complete telescope model and separated, is described.

2.1 MOST Model Description

The traditional design process for a complex system, such as a space telescope, involves the consideration of many design options in the conceptual design phase and the selection of a single point design for the preliminary design. Traditionally, detailed structural models are not created and analyzed until the preliminary design phase. However, if the design does not meet requirements once the analysis is complete, a costly redesign must be performed. Even if requirements are met, there will be no guarantee that the design chosen is optimal.

As new technologies are added to designs, it becomes difficult to make major decisions during the conceptual design phase, because the systems have no operational heritage. For these types of systems, it would be beneficial to evaluate more design options earlier in the design process. A parametric modeling approach applied early in conceptual design allows competing designs to be modeled and compared to one another. Such an approach allows for a large number of different architectures to be evaluated quickly and with a reasonable degree of fidelity. The designer can identify families of architectures which are likely to fulfill the system requirements and to have favorable attributes such as low mass and cost [32].

The MOST project has been developing such a parametric model for the design
of a large space telescope which utilizes advanced technologies. Given a set of input parameters, MATLAB is used to automatically create a structural finite element model. A normal modes analysis is then performed on this structural model using the Nastran finite element solver. An integrated state space model is created and analyzed using the DOCS (Disturbance, Optics, Controls, and Structures) integrated modeling toolbox [6]. Each architecture created can be evaluated using different analysis tools such as a dynamic disturbance analysis.

2.1.1 Parametric Model Generation

The goal of completely parameterizing a model is to allow the user to control both major and minor design choices. However, this makes developing the model significantly more challenging as there can be nearly limitless options for the final system architecture. The model must allow for major changes to the system architecture such as the primary mirror or secondary support tower type, in addition to allowing for changes to the dimensions of structural components or to material properties. Therefore, a completely parameterized model must include the ability to change both the values of numerical inputs and the basic geometry of the spacecraft.

To keep the information flow within the model clear, all inputs to the model are maintained in a separate parameters module. This module contains all design variables and material constants so that any user can quickly find and change variables in order to evaluate a new design configuration. In this way, changes can be made to the model by a user without full knowledge of each low level function. The parametric inputs will then flow from the top-level parameters module down to the lower level component creation modules as required. The input parameters are used to create the control system, finite element, disturbance, and optics models within separate modules. For such a complex and variable model to be created successfully, it is necessary to clearly define the relationships between individual model components. Figure 2-1 shows how the input parameters are passed through the modules. The parametric inputs are used to auto-generate a finite element model (FEM) of the spacecraft structure, as well as to create the disturbance and controller models which
are then added to the structural model.

In order to create the structural model, it is necessary to assume a basic architecture for the telescope. The spacecraft is designed to have a bus for general spacecraft operations such as communications and pointing. There is a separate optical telescope assembly (OTA) for observations. The bus is a tetrahedral frame which contains the reaction wheel assembly and other instruments. These non-structural components are represented using point masses; the masses of these components are determined using curve-fits based on historical data [34]. The OTA includes a primary mirror (PM), secondary mirror (SM), fast steering mirror (FSM), optics bench and secondary support tower (SST). In addition to each individual structural component, it is necessary to create connections between these model parts. Since the form of the structural components can change according to the input parameters, it is necessary to keep track of the correct attachment points in the module outputs. These connections are then created within separate modules. As shown in Figure 2-2, separate MATLAB modules exist to create the various design options for each of the structural components. An important feature of this type of model is that it can be extended to include new components by adding more modules.

Within this basic telescope architecture, the individual component geometries can
very greatly. For instance the primary mirror can be designed as a standard annular monolithic mirror by using the “Mono” module, but it can also be a segmented mirror with rib-stiffened hexagonal petals if the “Hex Isogrid” module is used. A completely different finite element model can be created by changing only a few of these inputs in the parameters file. Figure 2-3 shows two realizations of the MOST structural model. The telescope pictured on the left has a monolithic PM and a tripod SST, and the telescope on the right has a segmented PM and a hexapod SST. The mirror and SST type are the only two parameters which were altered in order to create these two very different models. The ease with which different telescope architectures can be created is an important feature of this model because it allows for large tradespaces to be generated quickly and automatically so that comparisons can be made.

Once the structural model has been generated for the chosen telescope architecture, a complete Nastran bulkdata structure is formed. The DOCS toolbox then generates an input file and calls Nastran to perform a normal modes analysis. The frequencies and modeshapes resulting from this analysis are then imported back into MATLAB.

Figure 2-2: Modules for Creation of Finite Element Model
2.1.2 Integrated Model

Using MATLAB, Nastran and the DOCS toolbox, an integrated model of the system is created as shown in Figure 2-4. Once the modes are imported into MATLAB, a state space system of the following form is created to represent the system dynamics.

\[
\begin{bmatrix}
\dot{q} \\
\dot{\dot{q}}
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-\Omega^2 & -2\zeta\Omega
\end{bmatrix}
\begin{bmatrix}
q \\
\dot{q}
\end{bmatrix}
\]  

(2.1)

where \( q \) represents the modal coordinates of the system, \( \Omega \) are the frequencies resulting from the Nastran analysis, and \( \zeta \) is a modal damping ratio which can be varied. In order to fully represent the behavior of this system, a disturbance model, isolators and controllers are added to this model.

The on-orbit dynamic disturbances to this model are due to the imbalances in the reaction wheel assembly which are used for attitude control and slewing. The size of the reaction wheels will scale with the inertia of the spacecraft in order to maintain the same slew rates, and it is assumed that the imbalance inertia is proportional to the wheel inertia, so the disturbances will also grow as the spacecraft becomes more massive. This disturbance model consists of harmonics of radial and axial forces at discrete wheel speeds. Disturbance contributions at multiple wheel speeds are com-
bined into power spectral density (PSD) curves representing the average disturbances across all frequencies. These can then be transformed to the spacecraft axes, and the disturbances from multiple wheels summed [16].

Isolators and controllers are added to the model as linear state space systems. Isolation is added to the system at two locations: between the reaction wheel assembly and the spacecraft bus and between the bus and the OTA. The isolators are modeled using low pass filters with variable corner frequencies. There are also up to four control systems for the spacecraft dynamics: an attitude control system (ACS), fast steering mirror control, dynamic piezoelectric wavefront control and petal control for the segmented mirrors. Various amounts of each of these types of control are used to create different control architectures for each telescope realization. These controllers are not the focus of this work, and are only mentioned to complete the description of the integrated model [8].

2.1.3 Performance Metrics

Once the model has been created, the reaction wheel disturbances are used to perform a frequency-based dynamic disturbance analysis in order to determine the optical performance of the telescope. The first optical performance metric for this telescope is the line-of-sight (LOS) jitter. The LOS jitter can be thought of as errors in the image quality due to motions of the entire OTA. The LOS jitter is approximated using rotations and translations of points on the primary, secondary and tertiary mirrors:
\begin{align*}
LOS_x &= -\frac{1}{f_1} \delta P_x + \frac{(M - 1)}{M f_1} \delta S_x + \frac{1}{M f_1} \delta T_x + 2\alpha P_x - \frac{2}{M + 1} \alpha S_x - \frac{2}{M + 1} \alpha T_x \\
LOS_y &= \frac{1}{f_1} \delta P_y - \frac{(M - 1)}{M f_1} \delta S_y - \frac{1}{M f_1} \delta T_y + 2\alpha P_y - \frac{2}{M + 1} \alpha S_y - \frac{2}{M + 1} \alpha T_y
\end{align*}

where \( f_1 \) is the focal length of the primary mirror, \( M \) is the secondary mirror magnification, \( \delta \) is translation and \( \alpha \) is rotation for points on the primary mirror (P), secondary mirror (S), and tertiary mirror (T).

A LOS jitter requirement has been determined based upon the angular resolution of the system. The amount of jitter in the system must be small enough so that light from a point source is not blurred between pixels on a camera. The total LOS error requirement is then equal to the angular resolution:

\[
LOS_{req} = \frac{1.22\lambda}{D}
\]

where \( \lambda \) is the wavelength of the imaged light and \( D \) is the primary mirror diameter; it was also assumed that dynamic disturbances account for 10% of the error. Therefore, for visible light (\( \lambda = 600 \text{ nm} \)) and a 3 meter diameter, the 3-\( \sigma \) value for \( LOS_{req} \) is 1.6 mas.

The other optical performance metric is the wavefront error (WFE), which represents the errors in the optical surfaces. In this model, the WFE is approximated based on the deformations of the primary mirror only \([1]\). Zernike polynomials are used to represent the primary mirror distortion. These are a sequence of polynomials in polar coordinates of radius, \( \rho \), and angle, \( \theta \), which are orthogonal on a unit circle. The Zernike equations are defined in Reference [35], and the first few shapes are shown in Figure 2-5. The out-of-plane displacements of the mirror surface are decomposed into coefficients of the first 48 Zernike functions. The root sum square of the weighted coefficients is then the WFE.
\[ WFE = \sqrt{\sum w_i^2 z_i^2} \]  

(2.4)

where \( z_i \) is the coefficient of Zernike term \( i \) and \( w_i \) is the weighting factor on this term.

The requirement for the WFE is:

\[ WFE \leq \frac{\lambda}{20} \]  

(2.5)

Again, it is assumed that this is a 3-\( \sigma \) requirement and the dynamic disturbances are allocated 10\% of the error budget. This results in a 1 nm requirement for the WFE.

<table>
<thead>
<tr>
<th>Tip/Tilt</th>
<th>Focus</th>
<th>Astigmatism</th>
<th>Coma</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho \cos \theta )</td>
<td>(-1+2\rho^2)</td>
<td>(\rho^2 \cos 2\theta)</td>
<td>(\rho(-2+3\rho^2) \cos \theta)</td>
</tr>
</tbody>
</table>

Figure 2-5: Zernike Shapes

In addition to these optical performance metrics, the telescope is judged on higher level system metrics. The first of these is the slew and settle time. Each time the spacecraft needs to rotate to point to a new location, vibrations will be introduced, and some amount of time will be required for the telescope to settle to within the LOS requirement. The telescope will be unable to image until it settles, and so it can be thought of as out-of-operations during the time of the slew and settling. A time domain simulation is used to calculate the settle time after a slew.

The final performance metrics for the system are mass and cost. The structural mass is computed directly using Nastran, and concentrated point masses are used to represent instruments in the bus which are not modeled in detail. These masses are sized to scale with the rest of the system. The cost metric is extremely difficult to
model for a space telescope which incorporates so many new technologies. Mass is 
used as an indicator of launch cost, but the mass cannot represent other costs such as 
manufacturing. In order to capture the cost of manufacturing, a relative cost model 
was developed based on the cost models which exist for many ground-based telescopes 
[29].

\[ C \propto n_{segs} D^{1.8} Z^{1.04} \]  

(2.6)

where \( n_{segs} \) is the number of segments in the primary mirror (one for a monolithic 
mirror), \( D \) is the diameter of a segment, and \( Z \) is the primary mirror sagitta, which 
is a function of the F\# and the conic constant. The \( Z^{1.04} \) factor captures the costs 
due to the polishing complexity which will vary according the the curvature of the 
mirror. This is purely a relative cost metric based on the manufacturing complexity 
of the mirror, and it will only be used for comparison between architectures. The 
LOS jitter, WFE, settle time, mass and cost are all used in the tradespace analysis 
to evaluate and compare many different telescope architectures.

### 2.2 Mirror Finite Element Model

A major component of this project is the structural model of a lightweight deformable 
primary mirror. Current state-of-the-art telescope designs have been developing primary 
mirrors with areal densities approaching 10 \( kg/m^2 \) (measured without including the 
actuators and supports) [20]. Therefore, a goal of the MOST project has been 
to model and analyze mirrors with areal densities in the range of 5 to 15 \( kg/m^2 \), 
while maintaining fundamental frequencies above launch vehicle requirements. Due 
to tightening resolution requirements, large primary mirrors are of particular interest, 
so the primary mirror diameter is varied from three to five meters. The large scale 
of these mirrors makes it especially hard to achieve acceptable stiffness in the mirror 
without adding mass. In order to achieve these conflicting goals of large diameter, 
low mass and high stiffness, this thesis focuses on the modeling and analysis of the 
primary mirror.
A model for a lightweighted mirror is developed using a rib-stiffened mirror geometry. The parametric model of this mirror is then used to study the effects of altering the geometry of the mirror and to optimize the structure for high stiffness to mass ratio. Since reduced stiffness, associated with lower areal densities, makes these mirrors more susceptible to thermal and dynamic disturbances, models of embedded actuators have been created in order to apply active controls which can improve the optical performance of the mirror. This section will discuss the modeling and finite element mesh generation of the primary mirror in greater detail. This model of the primary mirror will then be used, both as a part of the full MOST model and as a stand-alone model, to test the effects of parameter variation and to apply control laws.

2.2.1 Mirror Structure

As the diameter of the primary mirror continues to grow, the traditional monolithic mirror becomes increasingly difficult to launch using existing launch fairings. In addition, it was shown in Equation 2.6 that manufacturing cost scales with $D^{1.8}$. For this reason, a segmented hexagonal mirror was modeled as well as the traditional annular monolithic mirror. Visualizations of these two mirror types can be seen in Figure 2-6. A segmented mirror can be launched in a stowed configuration and deployed on orbit, and the six identical segments are much cheaper to manufacture than one large mirror. The segmented mirror consists of six hexagonal mirror petals which are rigidly attached to a central ring. The central ring is formed using bar elements, and each segment is cantilevered from this ring using rigid elements. Larger segmented systems can be created by adding a second ring of segments; however, six is the largest number of segments which is studied in this thesis. Separate MATLAB modules exist in order to generate these different mirror geometries, and the type of mirror generated is based upon a user input in the parameters file.

As mentioned previously, the mirror is modeled with a diameter between three and five meters with an areal density between 5 and 15 kg/m². This number for areal density does not include any actuators or support structure; for the segmented
mirror, it does not include the central ring or the attachments. The goal is for these lightweight mirrors to meet certain frequency requirements for launch. The entire primary mirror must have a fundamental frequency above 20 Hz and each segment of a segmented mirror must have a fundamental frequency above 100 Hz. In order to create a stiff mirror without adding a large amount of mass, a lightweighted mirror is modeled by incorporating rib-stiffening. As shown in Figure 2-7, the rib-stiffened mirror uses a triangular pattern of stiff ribs to support a thin face sheet which forms the optical surface. The tall, thin ribs on the back of the mirror create a very stiff structure without adding a large amount of mass.

Advanced materials are an important part of achieving the desired performance of the mirrors. Many current advanced telescope designs are considering using either
beryllium or silicon carbide for the primary mirror [17][18]. The important material properties are high stiffness to mass ratio, low coefficient of thermal expansion (CTE), high thermal conductivity and isotropic behavior. Stiffness to mass ratio is an important metric for dynamic performance, while a high conductivity to CTE ratio reduces the effect of thermal disturbances. Silicon carbide (SiC) is used throughout this project. It is a particularly good material choice due to its isotropic behavior, high stiffness to mass ratio, low coefficient of thermal expansion and high thermal conductivity. The material properties for SiC are shown in Table 2.1. One advantage of using a parametric model is that material properties for the mirror can be varied easily if there is interest in modeling a different material.

Table 2.1: Silicon Carbide Material Properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus, $E$</td>
<td>GPa</td>
<td>375</td>
</tr>
<tr>
<td>Density, $\rho$</td>
<td>$kg/m^3$</td>
<td>3200</td>
</tr>
<tr>
<td>Poisson’s Ratio, $\nu$</td>
<td>$kg/m^2$</td>
<td>0.17</td>
</tr>
<tr>
<td>CTE</td>
<td>ppm/$^\circ$C</td>
<td>2.44</td>
</tr>
<tr>
<td>Conductivity, $\kappa$</td>
<td>$W/m^2C$</td>
<td>157</td>
</tr>
</tbody>
</table>

The monolithic mirror is connected to the secondary support tower by kinematic bipod mounts at three points spaced 120 degrees apart in the $\theta$-direction; these support points are chosen to be at the intersection of the ribs. In Figure 2-7, the connection points are located at the three red dots. For the segmented mirror, the connection points are located at three points on the central ring. This kinematic mount fully constrains the six degrees of freedom of the mirror while allowing the mirror to stretch and shrink without warping. In the model, the three connection points are completely constrained in the vertical ($z$) and and circumferential ($\theta$) directions. Soft springs connect all other degrees of freedom, so that motions are not rigidly constrained. This system will non-redundantly constrain all six degrees of freedom, creating a kinematic mount. When the primary mirror is studied apart from the rest of the telescope structure, the kinematic mount is modeled by applying a single point constraint in the $z$ and $\theta$ directions only. By leaving the radial direction unconstrained, the mirror is free to expand and contract. In this way, the boundary conditions of a kinematic mount
are simulated for the mirror when it is analyzed apart from the telescope structure.

### 2.2.2 Mirror Mesh Auto-Generation

These structural mirror models are automatically generated by MATLAB functions. The inputs to these functions are the parametric inputs which define the mirror geometry, and the output is a bulkdata structure which is then used to create the finite element model in Nastran. This process is shown in Figure 2-8. Unlike most auto-mesh programs, the user does not need to interface with a graphical program. This allows the mirror finite element model to be created by a batch inputs file so that many geometries can be analyzed automatically.

<table>
<thead>
<tr>
<th>Inputs</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirror Type</td>
<td>Number Ribs</td>
</tr>
<tr>
<td>Diameter</td>
<td>Rib Aspect Ratio</td>
</tr>
<tr>
<td>Areal Density</td>
<td>Mass ratio</td>
</tr>
<tr>
<td>F#</td>
<td>Mesh Density</td>
</tr>
</tbody>
</table>

![Grid Point Generation](image1)

![Element Generation](image2)

**Nastran Bulkdata Structure**

BULKDATA =
CORD2C: [1x11 double]
GRID: [314x6 double]
CQUADR: [126x8 double]
CTRIA3: [216x5 double]
CBAR: [42x7 double]
MAT1: [2x6 double]

**Nastran Analysis**

Frequencies
Mode Shapes

Figure 2-8: Mirror Mesh Generation Method

Depending on the user input for the mirror type, a function is called to model either the monolithic or segmented hex mirror. The mirror diameter, number of ribs and mesh density parameters are then used in order to calculate the locations of the grid...
points in the mesh. For the monolithic mirror, grid points are generated across the mirror surface so that triangular elements can be formed, and both an inner and outer diameter are created in order to form the circular shape. For the segmented mirror, a hexagonal segment is generated with the grid points in triangular patterns. These grid points are then copied to new locations to generate the remaining five mirror segments. The mirror modeled is parabolic, and the values for diameter and F# are used to compute the curvature. The z-direction locations of the grid points are then determined based on this curvature.

Once the grid points have been generated, elements are formed by connecting the grid points. The mirror is modeled using two dimensional plate elements for the ribs and the facesheet (CQUADs and CTRIAs in Nastran). The user is able to specify the desired areal density (in kg/m²) of the mirror. In addition, the aspect ratio of the rib cross section and the percentage of mass in the mirror facesheet are all inputs to the model. These inputs are used in order to determine the height and thickness of the rib elements and the thickness of the facesheet elements.

Once the element definitions have been generated, the Nastran bulkdata structure is written. This structure includes coordinate systems, grid point definitions, element definitions, property cards for the elements, and material property information. The output of the mirror generation code is this bulkdata structure as well as any necessary information about connection points.

Figure 2-9: Variable Mesh Density for a One Meter Diameter Hexagonal Segment

In addition to the geometric parameters, the user can alter the finite element mesh density in order to improve the model fidelity. Figure 2-9 shows a one meter diameter
hexagonal segment with three rings of rib-stiffening and a varying mesh density. A ring of rib-stiffening is one hexagonal ring of rib elements. For instance, the mirror segment in Figure 2-9 has three rings of rib stiffening. The darker lines in this figure represent the rib structure and the lighter lines show the boundaries of each finite element. The user selects a desired number of elements per meter; however, the mesh generation code rounds this number so that there is a whole number of elements in a triangular rib cell. Figure 2-9 shows mirror meshes of (from left to right) 6, 18 and 30 elements per meter, which correspond respectively to 1, 9 and 25 triangular elements per cell.

Table 2.2: First 20 Frequencies for One Meter Hexagonal Segment with Varying Mesh

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>1 tri/cell</th>
<th>4 tri/cell</th>
<th>9 tri/cell</th>
<th>16 tri/cell</th>
<th>25 tri/cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>106.30</td>
<td>106.04</td>
<td>106.72</td>
<td>106.95</td>
<td>107.08</td>
</tr>
<tr>
<td>8</td>
<td>106.30</td>
<td>106.04</td>
<td>106.72</td>
<td>106.95</td>
<td>107.08</td>
</tr>
<tr>
<td>9</td>
<td>164.42</td>
<td>172.22</td>
<td>174.02</td>
<td>174.69</td>
<td>175.02</td>
</tr>
<tr>
<td>10</td>
<td>244.74</td>
<td>224.44</td>
<td>225.56</td>
<td>226.01</td>
<td>226.03</td>
</tr>
<tr>
<td>11</td>
<td>278.76</td>
<td>282.65</td>
<td>285.05</td>
<td>286.03</td>
<td>286.55</td>
</tr>
<tr>
<td>12</td>
<td>367.88</td>
<td>376.63</td>
<td>380.11</td>
<td>381.70</td>
<td>382.54</td>
</tr>
<tr>
<td>13</td>
<td>367.88</td>
<td>376.63</td>
<td>380.11</td>
<td>381.70</td>
<td>382.54</td>
</tr>
<tr>
<td>14</td>
<td>493.91</td>
<td>444.00</td>
<td>445.83</td>
<td>446.89</td>
<td>447.62</td>
</tr>
<tr>
<td>15</td>
<td>493.91</td>
<td>444.00</td>
<td>445.83</td>
<td>446.89</td>
<td>447.62</td>
</tr>
<tr>
<td>16</td>
<td>680.35</td>
<td>660.45</td>
<td>663.97</td>
<td>666.65</td>
<td>668.22</td>
</tr>
<tr>
<td>17</td>
<td>680.35</td>
<td>660.45</td>
<td>663.97</td>
<td>666.65</td>
<td>668.22</td>
</tr>
<tr>
<td>18</td>
<td>701.49</td>
<td>686.34</td>
<td>687.54</td>
<td>689.56</td>
<td>690.83</td>
</tr>
<tr>
<td>19</td>
<td>852.49</td>
<td>710.05</td>
<td>708.61</td>
<td>709.33</td>
<td>710.20</td>
</tr>
<tr>
<td>20</td>
<td>852.49</td>
<td>710.05</td>
<td>708.61</td>
<td>709.33</td>
<td>710.20</td>
</tr>
</tbody>
</table>

The effect of varying the finite element mesh was studied in order to determine what mesh fidelity is required to obtain good results. The single hexagonal segment shown in Figure 2-9 is used to test the effect of the mesh refinement on the fundamental frequencies of the mirror. Table 2.2 shows the first 20 frequencies of this mirror. Because this mirror was tested unconnected to the full telescope structure, the first
six frequencies represent rigid body modes and are equal to zero. To determine the amount of error caused by using a coarse mesh, the finest mesh model is considered to represent the exact solution. It can be seen that when only one element is used to represent a cell, the resulting frequencies have a large amount of error. For example, when there is one element per cell, modes 14 and 15 show a 10% error and modes 19 and 20 show an error of nearly 20%. If the mesh is increased to four elements per cell, the maximum error in these 20 modes is 2%, and if nine elements are used in a cell, the maximum error drops to 1%.

### 2.2.3 Piezoelectric Actuator Model

A goal of this project is to examine tradeoffs between lightweight structures and the use of active controls to minimize the effects of disturbances. In order to test the control laws, actuators must be included in the finite element model of the mirror. A surface-parallel-actuated mirror is created by embedding actuators in the ribs parallel to the mirror surface and away from the mirror facesheet, as shown in Figure 2-10. An axial strain is then induced in the actuators, causing a moment to be exerted on the mirror.

![Piezoelectric Actuator Embedded in Rib](image)

**Figure 2-10: Piezoelectric Actuator Embedded in Rib**

In Nastran, these actuators are modeled using CBAR elements connected to the rib elements using rigid RBAR elements, which have zero thermal conductivity. This
is necessary in order to thermally isolate the actuator from the remainder of the structure. Piezoelectric actuation is used for this mirror, although many other possibilities exist for actuating the mirror. Voltage is applied to the piezoelectric material, and a strain results according to Equation 2.7.

\[ \varepsilon = d_{33} \frac{V}{t} \quad (2.7) \]

where \( d_{33} \) is the piezoelectric constant, \( V \) is the applied voltage, and \( t \) is the thickness of the piezoelectric.

In order to model the piezoelectric effect in Nastran, a thermal analogy is used in which temperature is used to simulate voltage actuation, and the piezoelectric constant is represented by a coefficient of thermal expansion (CTE). However, when these actuators are used to correct the mirror shape under thermal disturbances, a problem arises. Now the CTE parameter must be used to represent the actual thermal expansion coefficient and also the piezoelectric constant [12]. This can be done by correctly scaling the input voltage.

\[ \varepsilon = \alpha T + d_{33} \frac{V}{t} \quad (2.8) \]
\[ \varepsilon = \alpha (T + \frac{d_{33} V}{\alpha t}) = \alpha T_{eff} \quad (2.9) \]

where \( \alpha \) is the CTE of the actuator, \( T \) is the temperature, and \( T_{eff} \) is the effective temperature which is applied in the finite element model to account for both the thermal and voltage effects on the actuator.

Although this actuator model, including the thermal representation of the piezoelectric effect, is not important for the dynamic analysis of the lightweight mirror, it is a major component of the active control system discussed in Chapters 3, 4 and 5.
2.3 Stiffness Optimization

The parametric mirror model, described in Section 2.2, is now used to determine what geometry will provide the best dynamic performance. In Section 2.3.1, the mirror will be studied apart from the rest of the telescope structure. Using this model, the homogeneous dynamics of the mirror are studied by varying the geometric parameters to determine the effect on stiffness. Then, in Section 2.3.2, a full disturbance to performance dynamic analysis is performed using the reaction wheel imbalances as the disturbance source. In this section, the effect of mirror parameters on the system performance metrics is determined.

2.3.1 Homogeneous Dynamic Analysis

The goal of the lightweight mirror design is to determine an optimal rib and facesheet geometry for a given areal density. Homogeneous dynamic performance is defined as best when the lowest flexible mode frequency is as high as possible. To determine what effect the different geometric parameters have on mirror stiffness, variations in parameters such as areal density, number of ribs, rib cross section and proportion in the facesheet will be analyzed using a three meter diameter monolithic mirror. The nominal parameters of this mirror are shown in Table 2.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirror Type</td>
<td>Monolithic</td>
</tr>
<tr>
<td>Diameter</td>
<td>3 meter</td>
</tr>
<tr>
<td>F#</td>
<td>1</td>
</tr>
<tr>
<td>Number Rib Rings</td>
<td>3</td>
</tr>
<tr>
<td>Facesheet Mass Ratio</td>
<td>50%</td>
</tr>
<tr>
<td>Rib Aspect Ratio</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2.3: Mirror Parameters for Geometry Variations

The first parameter which was varied is the areal density of the mirror. The areal density is defined as the mass of the mirror divided by the mirror area; this mass does not include the mass of any actuators or the support structure. Here, areal densities ranging from 5 to 30 kg/m² are tested. Two different rib geometries were used; one
mirror has three rings of ribs and one has six rings of ribs. As would be expected, Figure 2-11 shows that the first natural frequency of the mirror will increase as the areal density is increased. It can be seen in the figure that the 5 kg/m² mirrors do not meet the 20 Hz frequency requirement for either of these mirrors. This means that in order to create a very low areal density mirror which meets the launch frequency requirements, it will be necessary to alter other parameters.

Figure 2-11: Fundamental Frequency vs. Mirror Areal Density

In addition, it can be seen in Figure 2-11 that the mirror with three rib rings is stiffer than the mirror with six rib rings when both have the same areal density. In order to examine this effect further, the rib geometry on the back of the mirror is varied under the constraint of constant areal density. The density of the rib pattern can be altered by changing the number of hexagonal rings of ribs in the input file. The number of triangular rib cells across a diameter of the mirror is equal to twice the number of hexagonal rib rings. For a given areal density, and fraction of mass in the facesheet, a larger number of ribs means that these ribs will be smaller. The final limit of this is an infinite number of very small ribs which is the same as a non-rib-stiffened, uniform thickness mirror.

Figure 2-12 shows the first natural frequency of the lightweight mirror versus the
number of rib rings. Mirror areal densities ranging from 5 to 15 kg/m$^2$ were used in this analysis. Again, it can be seen that increasing the areal density will significantly improve the stiffness. It can be seen from this plot that a few large ribs on the back surface of the mirror provide more stiffness than a large number of smaller ribs. The “No Ribs” asymptotes labeled in the figure describe the fundamental frequency of the corresponding non-stiffened mirror of the same areal density; it is clear from these values that adding rib-stiffening significantly increases the stiffness of the large mirror.

![Figure 2-12: Fundamental Frequency vs. Number of Ribs](image)

Following this, the effect of changing the aspect ratio of the rib cross section is examined for a constant areal density and a constant number of ribs. The ribs always have a rectangular cross section, and the aspect ratio is defined as the height divided by the width of the rib ($h/t$), as pictured in Figure 2-10. Figure 2-13 shows that mirrors with tall, thin ribs are stiffer than those with short, wide ribs. The fundamental modes of the mirror are large scale bending modes, so adding mass to the ribs far away from the mirror’s centroidal plane will increase the area moment of inertia of the mirror. There is a practical limit to how much the aspect ratio of the ribs can be increased before buckling becomes a problem; however, this is purely a linear analysis, so potential buckling of the ribs is ignored. At some point the very
thin ribs would be at risk of buckling, and a more thorough non-linear analysis would be required.

![Figure 2-13: Fundamental Frequency vs. Rib Cross Section Aspect Ratio](image)

The final geometric parameter which was varied is the proportion of mass in the facesheet of the mirror. A low mass proportion means that the mass of the mirror is located almost entirely in the rib structure and the facesheet has become very thin, while a high mass proportion means that the ribs have become very small and the facesheet is thicker. Figure 2-14 shows the results of this variation. As more of the mirror mass is located in the ribs, the entire mirror becomes stiffer; this occurs because the ribs have a higher bending stiffness than the facesheet. However, below a mass proportion of 0.2, the facesheet has become so thin that low frequency modes occur in the facesheet structure within the cells. This optimal mass proportion will also be dependent on the number of ribs because this will change the size of the cells.

Therefore, to maximize the stiffness to mass ratio of the primary mirror, a design with only a few rings of rib-stiffening, high aspect ratio ribs and a low mass proportion in the facesheet. The specific mass proportion used will depend on the number of rings of ribs.
2.3.2 Dynamic Performance Analysis

Finally, the mirror model is connected to the OTA and spacecraft bus, and a tradespace analysis is performed in order to determine the effect of varying some of the key mirror parameters on the final performance metrics of the system. The parameters varied in this analysis are the mirror type, F#, areal density, rib aspect ratio and mass distribution between the ribs and facesheet. The values used for these parameters are shown in Table 2.4. It should be noted that this analysis does not include any active control of the mirror shape or hex petal positions to correct for WFE. A full factorial analysis was used to evaluate every combination of these parameters, which results in 360 possible telescope architectures whose performance is calculated. Each of these 360 designs requires the creation of a completely new finite element model. This means that an entirely new modal analysis must be run and the modes must be imported into Matlab. Although this is a complex process, the entire tradespace analysis was automatically performed on a desktop PC in approximately 24 hours.

The performance outputs from these designs are plotted against one another to determine trends in the model. Figure 2-15 shows the results of these 360 designs. The plots in the left column show the LOS plotted against the WFE and the plots in
Table 2.4: Parameters Varied in Tradespace Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirror Type</td>
<td>Mono or Hex</td>
<td>-</td>
</tr>
<tr>
<td>F#</td>
<td>1.0, 1.5, 2.0</td>
<td>-</td>
</tr>
<tr>
<td>Areal Density</td>
<td>5, 10, 15, 20, 25</td>
<td>kg/m²</td>
</tr>
<tr>
<td>Rib Aspect Ratio</td>
<td>2, 4, 8, 12</td>
<td>-</td>
</tr>
<tr>
<td>Facesheet Mass Proportion</td>
<td>25, 50, 75</td>
<td>%</td>
</tr>
</tbody>
</table>

The right column show the system mass plotted against the WFE. The plots in a given column are identical, but the symbols distinguish different parameter variations. The symbols in the plots in the first row indicate the effect of changing mirror type, and the symbols in the second row of plots reveal the effect of changing mirror areal density. The better performing designs lie in the lower left of each plot. The requirements for LOS jitter and WFE are shown as black lines on each plot.

Figure 2-15(a) shows that the monolithic mirrors tend to have better performance than the segmented mirrors. The pareto-optimal front can be examined in each of these figures to determine which parameters lead to favorable performance. The pareto front is defined as the set of designs where one metric can only be improved by degrading the other metric. A few of the designs on the pareto front have been selected, and are labeled in Figure 2-15(c) and Figure 2-15(d). The design parameters of these selected designs are shown in Table 2.5. By examining this table and Figure 2-15(a), it is seen that all of the optimal designs have monolithic mirrors. Since no active control of the mirror was used, this result could have been expected. The monolithic mirrors have a natural shell stiffening, while the segmented mirrors have flexible petal flapping modes due to the connections between the segments and the central ring. This low frequency flapping mode results in the poor optical performance which can be seen in these plots. Both Figure 2-15(a) and Figure 2-15(b) show that very few of the systems with segmented mirrors are able to meet the 1 nm WFE requirement. In fact, when Figures 2-15(d) and 2-15(b) are examined, it can be seen that the segmented systems which meet the WFE requirement have areal densities of 15 kg/m² or greater. Therefore, it will be necessary to introduce active controls if lightweight, segmented mirrors are desired in the final design.

50
Figures 2-15(c) and 2-15(d) can be studied in order to determine the effect of changing the primary mirror areal density. Obviously, as areal density increases, so
does the total system mass. The OTA becomes heavier due to the increased mirror mass, and the bus will become heavier because the point masses in the bus, which represent instruments, are scaled according to the mass of the OTA. Also, as areal density is increased, more designs meet both the LOS and WFE requirements. However, there are a few designs with areal density less than 15 kg/m$^2$ which meet the requirements. These designs are all monolithic mirrors with $F\# = 1$. These highly curved monolithic mirrors have enough natural shell stiffening to meet the performance requirements without the addition of active controls. However, Equation 2.6 showed that this type of mirror will be the most expensive to manufacture.

![Figure 2-16: Tradespace Results Showing Variation in Rib Aspect Ratio](image)

Changing the rib aspect ratio has a much smaller effect on the final performance than many of the other parameters. By examining Figure 2-16, it can be seen that, for a given design, increasing the aspect ratio of the ribs will improve the performance slightly. This plot shows many vertical groupings of designs for which the rib aspect ratio is the only changing variable. Each of these groupings has a constant mass, and the WFE is reduced as the aspect ratio is increased. Although it is not a large effect, this shows that the rib aspect ratio is one parameter which can be altered to improve
the mirror without adding mass to the system. As mentioned in Section 2.3.1, this analysis does not include the buckling of ribs which will limit the cross section aspect ratio.

2.4 Conclusions from Dynamics Model

The parametric model created for the MOST project is an extremely useful tool for evaluating the performance of advanced space telescopes by evaluating the effect on performance of varying design parameters. Favorable families of architectures can be identified early in the design process by using an automatic tradespace analysis. This type of model allows a user to investigate the roles of many variables in influencing final system performance metrics. Investigating the design space early can ensure that an appropriate design is chosen for the detailed modeling which will occur during the preliminary design phase.

In particular, the lightweight mirror was studied for dynamic performance, both as a separate component and within the entire system. It was discovered that, for a constant areal density mirror, a stiffer mirror can be created by using a few large ribs instead of many small ribs. Further work on the rib geometry will need to incorporate the effects of launch loads. In addition, a full tradespace disturbance analysis was run, and it was seen that the traditional monolithic mirrors with high areal density provide superior optical performance. However, lightweight and segmented systems have many favorable attributes when factors such as mass, manufacturing costs and launch fairing restrictions are considered. This suggests that actively controlled mirrors will be necessary in order to compensate for the inferior performance of the lightweight systems. The next chapters will discuss the development of just such an active mirror; in this case the control will be developed for the mitigation of thermal effects.
Chapter 3

Mirror Shape Control with Embedded Sensors

The previous chapter described the development of the MOST model, which was created to analyze errors caused by on-orbit dynamic disturbances. There are many other sources of disturbances which were not considered in this model. In particular, quasi-static changes in the thermal environment are another source of errors in the primary mirror. These errors can be corrected by controlling the shape of the primary mirror using the embedded actuators which were described in the previous chapter. As described in Chapter 2, the WFE must be corrected to less than $\frac{\lambda}{20}$. The thermal disturbances are allocated 20% of the error budget. This results in a 3-σ requirement of 2 nm for the WFE for a telescope operating in the visible spectrum. This requirement can be used to evaluate the effectiveness of the shape control algorithms at correcting various thermal disturbances.

In this thesis, the goal is to control the shape of the primary mirror using only sensors embedded in the mirror instead of a traditional wavefront sensor. Wavefront sensors are very expensive, and they cause some of the light to be lost. In addition, errors could occur due to the separation of the mirror and the sensor [15]. Strain gages and temperature sensors will be studied to determine which is more promising. It will be necessary to determine if these sensors have the required resolution. The embedded sensors must be able to distinguish the different distortions which result
from varying temperature distributions.

This chapter will present the thermal disturbance models which are used to test the shape control algorithms. The response of the mirror to these disturbances will be examined. In Section 3.2, a simple, representative structure will be examined to analyze the distortions which result from expected temperature distributions. A one-dimensional beam will be used as the representative structure. Following this, in Section 3.3, a simplified model of a surface parallel actuated mirror is created; this model is a flat plate with actuators raised off the back surface on posts. This model is used to develop and analyze the shape control algorithms.

3.1 Thermal Disturbance Models

The primary mirror of a space telescope can experience thermal disturbances due to environmental effects and internal heat sources. In addition, the thermal environment during manufacture, assembly and testing in the laboratory may be different from the operating environment. Due to the very high precision required for the optical surface, even small changes in the temperature may cause problems in the mirror. As mentioned in Chapter 2, silicon carbide (SiC) is used for the mirror material. It was chosen partially for its high ratio of thermal conductivity to CTE [17]. This means that high thermal gradients will not be maintained in the mirror, and the mirror material will have a low expansion with changes in temperature.

The simplest temperature gradient which is studied is a bulk temperature change of the entire mirror. In addition, a linear and an exponential temperature gradient are used to represent the effect of a heat source located at one edge of the mirror. A conical shaped gradient is used to represent the effect of a heat source located at the center of the mirror. Each of these disturbances has a peak temperature change of 1°C, but the model is linear so the control correction factors will scale. Plots of these temperature distributions on a circular mirror are shown in Figure 3-1.

Temperature gradients through the thickness of the mirror have the ability to create the largest distortions in the optical surface. However, the high conductivity
of the SiC diminishes the problem of through-thickness gradients. The heat transfer equation can be used to determine what heat flux would be necessary to cause a 2 nm disturbance of the mirror surface:

$$\dot{q} = -k \frac{dT}{dx} = -k \frac{\Delta T}{\Delta x}$$  \hspace{1cm} (3.1)

where $\dot{q}$ is the heat flux per area, $k$ is the conductivity of the material, $\Delta T$ is the difference in temperature and $\Delta x$ is the thickness of the material. It was determined, using a finite element model, that a through thickness $\Delta T$ of approximately 0.001°C will cause a WFE of 2nm. The mirror thickness for this model is 2.5mm. The thermal conductivity of silicon carbide is approximately 157 W/m°C. This results in a heat flux of 62.8 W/m², which would be an high number to maintain within the mirror for any length of time. Because of the high conductivity, through-thickness gradients
will quickly equalize, and a bulk temperature change in the mirror will result. Based on this result, it was decided to ignore through-thickness temperature gradients for the simplified flat plate mirror model.

At this time no thermal modeling program has been linked to the finite element solver. Therefore, the temperature distributions are implemented in the finite element model using point temperatures located at each of the nodes in the model. This is then treated as a loading condition in Nastran. Temperatures within the elements are determined by interpolation between connected grid points. The effects of these thermal disturbances will initially be analyzed using a one-dimensional beam model. The insight gained from the beam model can then be applied to a more complex plate model.

### 3.2 Simple One-Dimensional Beam

A simple beam model is used to begin examining the expected distortions due to the thermal loads. This can then be used to determine the necessary sensor locations. The beam is a one-dimensional (1-D) approximation of the plate model which will later be used to develop a control algorithm. The mirror is supported using kinematic mounts which allow for expansion in the radial direction; in order to simulate this on the beam, a simply supported boundary condition is used. This way, the beam can freely expand in the axial direction. A schematic of the beam model used is shown in Figure 3-2.

![Figure 3-2: Simple Beam Model](image)
The beam pictured on the right has a surface parallel actuator mounted to the top surface. This will affect the beam distortions because the coefficient of thermal expansion (CTE) of the actuator is different from the CTE of the beam, which is the case for the surface parallel actuated mirror.

3.2.1 Beam with Bulk Temperature Change

The simplest temperature disturbance considered is a bulk temperature change, where the entire structure changes temperature by a uniform amount. The structure will deform according to the strain equation:

$$\varepsilon = \alpha (T - T_0)$$  \hspace{1cm} (3.2)

When there is no actuator attached, the beam will expand freely and equally in all directions. The strain will be equal on the top and bottom surfaces of the beam. As a result, there will be no curvature. However, when an actuator with a CTE smaller than the mirror CTE is attached to the beam, it mechanically prevents the top surface of the beam from expanding, and a curvature is induced in the beam. Figure 3-3 shows this effect on a simple finite element model. In this model, the beam and the actuator are formed using simple beam elements with temperatures applied at the nodes. When a full plate model is used with actuators across the entire surface, the bulk temperature change will create a cupping shape, which is the 2-D equivalent to the beam deformation.

![Figure 3-3: Beam Model with Bulk Temperature Change](image)

The effect of the CTE mismatch between the mirror material and the actuators is important to understand before analyzing a full mirror model so that bulk tem-
perature changes are not ignored. The more complex temperature disturbances will cause deformations of the beam even without the presence of actuators, and will be analyzed next.

3.2.2 Beam with Through-Thickness Temperature Gradient

Although it is unlikely to occur in a SiC mirror, the beam model can be used to analyze the effects of a through-thickness temperature gradient. If the the top and bottom surfaces of the beam are at different temperatures, a through thickness gradient occurs, and the top and bottom surfaces of the beam will expand by different amounts, causing the beam to bend. Each surface has a strain according to Equation 3.2; however, this will result in an overall curvature of the beam. The curvature can be quantified based on the temperature difference.

\[ \frac{T}{2} \int_{-T/2}^{T/2} t \, dx \]

Figure 3-4: Cross Section of Beam with Thermal Gradient

The cross section of a beam with a through-thickness gradient is shown in Figure 3-4. Using this figure, the strain on the top and bottom surfaces of the beam can be written as:

\[ \varepsilon_{\text{top}} = \frac{\alpha T}{2} \]

\[ \varepsilon_{\text{bot}} = -\frac{\alpha T}{2} \]
The curvature can now be calculated by simple geometry. A small angle approximation is used to simplify the problem. This is a valid approximation since the out of plane displacements are generally much smaller than the diameter of the mirror.

\[
\begin{align*}
    t d\theta &= \varepsilon_{\text{bot}} dx - \varepsilon_{\text{top}} dx \\
    \kappa &= \frac{d\theta}{dx} = -\frac{\alpha T}{t}
\end{align*}
\]  

(3.5)  

(3.6)

where \( t \) is the beam thickness and \( dx \) is the beam width as shown in Figure 3-4. Therefore, the curvature in the beam with a through thickness gradient is a function of the temperature on each surface (\( T \)), the CTE of the material (\( \alpha \)), and the thickness of the beam (\( t \)). This effect is tested on a simple finite element model as shown in Figure 3-5. For this model, it was necessary to use solid elements so that different temperatures could be applied on each surface. In addition, the strain can be read on each surface of the solid elements. The strains are shown by the color map in Figure 3-5.

![Figure 3-5: Simple Beam with Through Thickness Thermal Gradient](image)

The results of the one-dimensional model can now be used to determine where sensors must be located in order to obtain information about the curvature of the beam.
3.2.3 Beam with Linear Temperature Gradient

Variations in the thermal distributions across the mirror surface will also be examined. This type of disturbance will be examined by applying a linear temperature gradient across the length of the simple beam. A temperature gradient from +1°C to -1°C is applied to the same beam used in Section 3.2.2. This is shown in Figure 3-6. This beam is constrained at the end points as shown in the figure.

![Figure 3-6: Simple Beam with Linear Temperature Gradient](image)

Each element in the beam will have strains according to Equation 3.2. It can be seen in Figure 3-6 that where the temperature is negative, the beam displacement will be negative. As the temperature rises above zero, the beam displacement becomes positive. Therefore, this type of temperature distribution causes the beam to curve, even with no actuators present.

3.2.4 Sensor Locations

The bulk and through thickness gradient cases demonstrate the need for strain gages at two heights through the mirror thickness. In order to measure a curvature, it is necessary to find the difference between two strain values separated through the thickness. Due to the CTE mismatch effect between the actuators and the mirror, even the bulk temperature change will induce a curvature in the mirror which can only be measured by two strain gages. In addition, it may be necessary to include
temperature sensors along with the strain gages because the resistance of a strain gage is temperature dependent.

Another option is to use only temperature sensors. In this case, it is still necessary to place sensors at two heights in order to distinguish between bulk and through thickness temperature changes. However, as discussed in Section 3.1, it is possible to ignore the through thickness gradients. If through thickness gradients are ignored, only one layer of temperature sensors would be necessary to measure the errors in the mirror surface.

### 3.3 Plate Model for Deformable Mirror

Following the analysis of the one-dimensional structure, a simple surface parallel actuated mirror is modeled in order to test the effects of thermal disturbances and to create the control laws. This is not the full rib-stiffened mirror, described in Chapter 2. However, this simple mirror serves as a starting point to begin evaluating the controls using embedded sensors.

#### 3.3.1 Finite Element Model

The simple model which is used to develop the control law is a flat mirror with no rib-stiffening. The discrete surface-parallel actuators are mounted on posts and offset from the back surface of the mirror as shown in Figure 3-7. In this way, as the actuators expand, a moment is induced at the mirror surface. A spacer is used between the actuator and the post elements, so that the actuators can maintain a specific length regardless of the number and layout on the mirror surface. For this mirror, the spacers are modeled as bars with a CTE of zero.

This finite element model for is created in Nastran. The mirror is formed using three-dimensional triangular solid elements (CPENTAs). Solid elements are necessary for this model because temperatures are applied to both surfaces. Also, displacements are read out of both the optical and back surface of the mirror to get the strain at both surfaces and, as a result, curvature. The actuators and posts are bar elements.
The connections between these bar elements and the solid elements which form the mirror surface can be complex. Since solid elements, as used in the mirror, have no stiffness in the three rotational directions, a bar connected to a solid element will act like a “ball and socket” joint and, unfortunately, no moments will be transmitted to the solid element when the bar element displaces. To remedy this, RBE3 elements are placed inside the solid elements. The RBE3 elements are interpolation elements which can be used to compute the rotation of the attachment point based on the translations of adjacent points. This transmits the loading to the independent degrees of freedom.

The actuators on this mirror are laid out in hexagonal patterns, and models exist for various numbers of actuators, as shown in Figure 3-8. A clear aperture is defined by the blue circle whose diameter is slightly smaller than the diameter of the mirror. All of the control laws developed are designed to minimize distortions within the clear aperture because it is difficult to design a circular mirror with a hexagonal actuation geometry to the edges. The mesh density is varied slightly as the number of actuators
is changed so that the actuators fill the clear aperture. As the number of actuators and mesh density are varied, all other parameters are held constant. The geometric parameters for this flat plate mirror are shown in Table 3.1.

Table 3.1: Flat Plate Mirror Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>40 cm</td>
</tr>
<tr>
<td>Clear Aperture</td>
<td>35 cm</td>
</tr>
<tr>
<td>Mirror thickness</td>
<td>2.5 mm</td>
</tr>
<tr>
<td>Actuator Height</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Actuator Length</td>
<td>2.0 cm</td>
</tr>
<tr>
<td>Actuator Radius</td>
<td>1.5 mm</td>
</tr>
</tbody>
</table>

This flat plate mirror model is used to develop and analyze the control using embedded sensors. Therefore, it is necessary to develop models of both strain gage and temperature sensors within the finite element model.

3.3.2 Sensor Models

The two types of embedded sensors which are modeled are strain gages and temperature sensors. Temperature sensors can easily be modeled by directly reading out temperature values from the Nastran model. If the temperature sensor is located at a FEM node, the corresponding nodal temperature can be output. If the desired sensor location is somewhere other than a grid point, then the output of the temperature sensor is a weighted average of the temperature at the nearby grid points.

Strain gages are located on the mirror at two different heights through the mirror thickness in order to measure the local curvature of the mirror surface. Curvature is defined as:

\[
\kappa = \frac{\varepsilon_{\text{top}} - \varepsilon_{\text{bot}}}{\Delta z}
\]  

(3.7)

where \( \varepsilon \) is the measured strain and \( \Delta z \) is the distance between the two strain gages. In order to obtain the strain measurements from the finite element model, the displacements in the global x and y directions (as shown in Figure 3-9) at each node
are output. Using these, the change in length of each triangular element side can be
determined, and the strain is computed according to:

\[ \varepsilon = \frac{\Delta L}{L} \]  

(3.8)

In addition, the angle of each triangle is stored so that the strains can be transformed
from the angle of the element into the global x and y directions using a transformation
matrix. For the baseline control law, the strain gages are assumed to be located below
the actuators, and they are generally aligned and perpendicular to the actuator as
shown in Figure 3-9.

![Figure 3-9: Locations of Strain Gages (Back Surface Only Shown)](image)

It is then possible to obtain either the temperature or strain from the finite element
model at the locations of the sensors to simulate embedded sensing. However, real
sensors will have a limited resolution. These limits should be examined further to
determine whether embedded strain gages or temperature sensors will actually be
able to control the mirror surface to the required surface error.

### 3.4 Sensor Sensitivity Issues

Before developing control algorithms, the sensors will be studied to determine if they
will have a fine enough resolution to correct the mirror to the required surface error.
of $\lambda/20$. To begin, the resolution of the strain gage sensors is examined. To measure the local curvature, it is necessary to resolve the difference between the strain measured by strain gages at two different locations through the thickness of the mirror. A given amount of error in the mirror surface could be caused by either a high spatial frequency deformation or by a low spatial frequency deformation. Two strain gages separated through the thickness of the mirror will have the most difficulty resolving the strain due to a small amount of curvature. Therefore, errors in the surface due to a low frequency deformation, such as the focus mode, will be the most difficult to measure using strain gages. Here, this focus mode is approximated as a sinusoid of the form:

$$w = w_1 \sin \left( \frac{\pi x}{L} \right)$$  \hfill (3.9)

where $w_1$ is the amplitude of the deformation and $L$ is analogous to the diameter of the mirror. As a result, there is half a wave across the diameter, similar to the focus shape. The curvature is obtained from this sinusoid by:

$$\kappa_x = \frac{\delta^2 w}{\delta x^2} = -w_1 \frac{\pi^2}{L^2} \sin \left( \frac{\pi x}{L} \right)$$  \hfill (3.10)

This curvature is then related to the surface error requirement where:

$$\sigma_w = \frac{\lambda}{n}$$  \hfill (3.11)

and,

$$\sigma_w^2 = \lim_{X \to \infty} \frac{1}{X} \int_0^X w^2 dx$$  \hfill (3.12)

$$\sigma_w^2 = \lim_{X \to \infty} \frac{1}{X} \int_0^X w_1^2 \sin^2 \frac{\pi x}{L} dx$$  \hfill (3.13)

$$\sigma_w^2 = \frac{w_1^2}{2}$$  \hfill (3.14)
This amplitude can then be substituted into Equation 3.10, and related to the strain using Equation 3.7. In order to control the mirror shape to $\lambda / 20$, the strain gages must be able to resolve strain at these levels.

$$w_1 = \frac{\lambda \sqrt{2}}{n}$$  \hspace{1cm} (3.15)

It can then be seen that the necessary strain resolution depends on $n$, the surface error requirement factor, $L$, the diameter of the mirror, and $\Delta z$, the distance between the top and bottom strain gages. These parameters, as well as the locations of the sensors on the mirror, can be altered to improve strain measurements.

For example, if the mirror has a 40 cm diameter, a thickness of 2.5 mm with a WFE requirement of $\lambda / 20$, the required strain gage resolution is 0.0065 $\mu$strain. However, this assumes that the strain gages have been placed on both surfaces of the mirror, which is not possible, since it would contaminate the optical surface. The theoretical limit for the resolution of a strain gage due to the residual Johnson noise in the resistor is 0.0012 $\mu$strain [21]. Existing gages have been tested, and noise levels have been measured at approximately 0.02 $\mu$strain. Temporal averaging can be used to improve the resolution of these gages at the expense of introducing time delay into the control. This analysis was performed for a structure with no actuators. The presence of actuators on the mirror surface will introduce local curvature, making it easily to measure the errors in the mirror.

If temperature sensors are used instead of strain gages, the resolution should not be a major issue. The finite element model was used to determine what value of $\Delta T$ was necessary to cause a surface error of $\lambda / 20$. It was found that a bulk temperature change of 0.02°C will create this amount of error. Current temperature sensors have resolutions to the level of 0.005°C, so it should be possible to use temperature sensors.
to measure the error in the mirror to necessary levels.

### 3.5 Conclusions

This chapter presented the thermal disturbance models which will be used to analyze the shape control. To begin, a simple, one-dimensional structure was used to analyze the expected response to these disturbances. It was shown that a bulk temperature change will result in curvature of a beam when an actuator with a different CTE is attached to the beam. Following this, thermal gradients through the thickness and across the beam were applied. Both of these disturbances resulted in distortions of the structure even without a surface-parallel-mounted actuator with a mismatched CTE. However, for the thin SiC mirrors studied in this thesis, through-thickness thermal gradients will not be important. Through-thickness gradients will quickly equalize because of the high conductivity of SiC.

A flat plate mirror model was created in order to develop the shape control algorithms. The flat plate is formed using three-dimensional triangular elements with surface-parallel actuators mounted on the back side of the mirror using post elements. The actuators are mounted in hexagonal patterns, and the number of actuators used is variable. Therefore, the effect of using more actuators to control the mirror can be evaluated. This model is much smaller than the full rib-stiffened mirror presented in Chapter 2, and so it will be used as a representative structure for the analysis of shape control.

Before beginning the development of shape control algorithms, both types of sensors are evaluated for resolution requirements. It was determined that temperature sensors will have the required resolution to correct the mirror to the required surface error. If strain gages are used, it may be necessary to use temporal averaging to improve the resolution of existing sensors.
Chapter 4

Static Shape Control Algorithms for WFE Minimization

The goal of this shape control problem is to correct the surface of the primary mirror using only embedded sensors and actuators. Surface-parallel-mounted piezoelectric actuators are used to alter the mirror shape. Control laws are developed for both embedded strain gages and temperature sensors in order to determine if either of these sensors can be used to control the mirror shape to the required precision. The traditional method of shape control for deformable mirrors is to directly measure the WFE in the mirror using a wavefront sensor such as a Shack-Hartmann sensor which is in the optical path somewhere removed from the primary mirror [15]. Embedded sensors could potentially remove the cost and complexity of including a dedicated wavefront sensor in the optical system.

This chapter will describe the development of the control laws which use embedded sensors to minimize errors in the mirror surface. Here, it is assumed that the only disturbances being corrected are caused by thermal effects that are slowly time-varying. To begin, a traditional control algorithm, utilizing a wavefront sensor will be presented. Then control algorithms for both the embedded strain gages and temperature sensors will be developed. Control laws using both full feedback from every element in the model and feedback from a limited number of sensors will be developed. Both of the control laws will be evaluated on the simplified mirror model.
which was described in Chapter 3 for a simple case of a bulk temperature change in order to determine how well the control laws correct the WFE of the mirror.

4.1 Traditional Shape Control

A traditional active optical system includes a deformable mirror, wavefront sensor and control system. This type of system is shown in Figure 4-1. This figure shows light hitting a deformable primary mirror. Alternatively, it would be possible to use a small deformable mirror somewhere else in the optical path, instead of actuating the primary mirror directly. After hitting the deformable mirror, the light is directed into a beamsplitter which divides the light. Some of the light is then used to form an image while the remainder of the light is directed into a wavefront sensor, which measures the errors in the wavefront.

![Figure 4-1: System Diagram for Traditional Active Optical Control](image)

The measurements from the wavefront sensor are then sent to a control system which computes the inputs to the actuators on the deformable mirror in order to correct the wavefront. The control of the deformable mirror is generally based on knowledge of the actuator influence functions. Actuator influence functions are used to define
the relationship between the actuator commands and the deformations of the mirror surface. Usually, a least squares method is used to determine the set of actuator commands which will minimize the error in the mirror surface. Mirror deformations due to each actuator command multiplied by the influence of that actuator are added to the initial distorted mirror surface, which results in the corrected mirror shape. Figure 4-2 shows this process of correcting a deformed mirror surface based on the actuator influence functions. It can be seen in this figure that the corrected mirror shape has smaller deformations (two orders of magnitude smaller) which are of higher spatial frequency than the initial mirror surface.

Figure 4-2: Correction of Distorted Mirror by Adding Actuator Influence Functions Scaled by Input Values

Influence functions can be generated experimentally, but for the simulation in this thesis they are created using the finite element model. Each actuator influence function is generated by commanding a known input on one actuator and zero input into all other actuators. The resulting shape of the mirror is then output and decomposed
into the Zernike coefficients; these Zernike coefficients then become one column of the actuator influence matrix. These coefficients are computed using:

$$z = A \times c$$ \hspace{2cm} (4.1)

where \(z\) is a vector of the z-displacements of the nodes on the optical surface, \(A\) is a matrix of the Zernike polynomials discretized over the mirror surface, and \(c\) is a vector of the Zernike coefficients. Given the z-displacements, the desired coefficients are computed using Gaussian elimination.

Each actuator in the mirror is considered to be independent, so that the final shape of the mirror is described by a linear sum of each influence function:

$$W = \sum_{i=1}^{N} \Sigma_i u_i$$ \hspace{2cm} (4.2)

where \(\Sigma_i\) is the \(i^{th}\) column of the actuator influence matrix, and \(u_i\) is the input to actuator \(i\). A problem occurs with this assumption of independence because there is some cross-coupling between the actuators. The amount of cross-coupling depends on the ratio of actuator stiffness to the faceplate stiffness [30]. This cross-coupling will introduce some amount of error into the control of the mirror.

Influence functions traditionally relate inputs to the surface deformations of the mirror; however, it is also possible to generate influence functions which correspond to the strain across the mirror surface. The resulting shape of the mirror under actuation can be used to compute the strains in the mirror surface, and the strains are the column of the influence matrix. When a wavefront sensor is not used, it is also necessary to create a disturbance influence function which relates the temperature disturbances to changes in the mirror shape. This is necessary because the changes in mirror shape cannot be obtained from the sensors directly as with a wavefront sensor. This disturbance influence function matrix is generated using the finite element model by the same process as the actuator influence functions.
4.2 Control Using Strain Gage Sensors

When strain gage sensors are used, a control law is developed which minimizes the curvature in the clear aperture of the mirror surface. This will have the effect of minimizing the out-of-plane displacements, and therefore the WFE. Implementing curvature minimization is simpler than attempting to minimize the WFE directly. The curvature is directly related to the strains measured by the sensors. The strain in the mirror is a function of the actuator inputs and the thermal disturbances, and the curvature in each element of the mirror is then computed based on Equation 3.7. The strain is induced both by the actuator commands and the thermal disturbances. In order to compute the strain the actuator influence function, Σ, and disturbance influence function, Γ, are used.

\[ \varepsilon = \Sigma u + \Gamma T \]  

(4.3)

Curvature can be calculated from the strain using Equation 3.7. Here, \( \Phi \) is a matrix which computes curvature from strain for every element in the model.

\[ \kappa = \Phi\varepsilon = \Phi\Sigma u + \Phi\Gamma T \]  

(4.4)

The strain gages measure a subset of the strains in the mirror. For this problem, the sensor, represented by \( y \), are the strains measured by the strain gages. The matrix \( P \) is used to select the locations of the strain gages, and \( \varepsilon \) is the entire set of strains in the finite element model. For the baseline controller, the number of strain gages is equal to four times the number of actuators because it is necessary to place strain gages at two heights to compute curvature, and the strain gages are located underneath the actuators in both aligned and perpendicular directions. The \( P \) matrix locates the gages and calculates the strain for the correct direction.

\[ y = P\varepsilon \]  

(4.5)

The control feeds these sensor outputs to the piezoelectric actuators through a static gain matrix. This is a feedback problem, so that the actuator commands \( u \) are some
function of the sensor $y$. The feedback matrix, $F$, must be determined so that the error in the clear aperture is minimized.

$$u = F y$$  \hfill (4.6)

In this section, two methods for determining the actuator inputs will be shown. First, a full feedback problem, which utilizes strain feedback from every element in the model, will be formulated. Following this, a control algorithm which uses feedback from a limited number of sensors will be developed.

4.2.1 Full Feedback Problem

It is simplest to begin this control problem by examining the control problem with full feedback from every element in the model. For the full feedback problem, $\kappa^T \kappa$ will be directly minimized over the the unknown actuator inputs. The feedback matrix, $F$, does not appear in this expression because for this case feedback from the sensor outputs are not used. Instead, the curvature is described by the influence functions, $\Phi$ matrix and thermal disturbances.

$$J = \kappa^T \kappa = (T^T \Gamma^T \Phi^T + u^T \Sigma^T \Phi^T)(\Phi \Sigma u + \Phi \Gamma T)$$  \hfill (4.7)

Then, take the derivative of $J$ and set it equal to zero in order to solve for the set of actuator inputs, $u$.

$$\frac{\delta J}{\delta u} = 2\Sigma^T \Phi^T \Phi \Sigma u + 2\Sigma^T \Phi^T \Phi \Gamma T = 0$$  \hfill (4.8)

$$u = -(\Sigma^T \Phi^T \Phi \Sigma)^{-1} \Sigma^T \Phi^T \Phi \Gamma T$$  \hfill (4.9)

Now check the second derivative of $J$ to make sure that this is a minimum point.

$$\frac{\delta^2 K}{\delta u^2} = 2\Sigma^T \Phi^T \Phi \Sigma$$  \hfill (4.10)
The second derivative is positive semi-definite, so this solution is a minimum point. This solution for \( u \) can be applied to the model in order to determine how well the full feedback control law will work. However, when there is a limited number of sensors available, the full feedback algorithm cannot be used, so another control algorithm is developed.

**4.2.2 Output Feedback Problem - Numerical Optimization**

In general, the number of sensors available will be limited, so it is not possible to use the full feedback algorithm developed previously. Therefore, this section describes the formulation of an output feedback problem. Inputs to the actuators are a linear function of the sensor outputs, as shown in Equation 4.6, which in this case are the strains measured by the strain gages (Equation 4.5). By combining Equations 4.3 to 4.6, \( \kappa^T \kappa \) can be rewritten as:

\[
J = \kappa^T \kappa = T^T \Gamma^T (I - \Sigma FP)^{-T} \Phi^T \Phi (I - \Sigma FP)^{-1} \Gamma T
\]  
(4.11)

\[
\frac{\delta J}{\delta F} = 2 (\Sigma^T (I - \Sigma FP)^{-1} \Gamma T)^T \Gamma (I - \Sigma FP)^{-1} \Gamma T \Phi^T \Phi (I - \Sigma FP)^{-T} P^T
\]  
(4.12)

This equation is too complex to be solved analytically for \( F \), so numerical optimization is used to minimize the curvature. This equation is particularly difficult to solve for \( F \) because of the \((I - \Sigma FP)^{-1}\) terms. These terms exist because changes in both temperature and actuator inputs will change the curvature.

The Matlab function \( fminunc.m \) is used to perform an unconstrained, nonlinear optimization. The function to be minimized is \( \kappa^T \kappa \) as defined in Equation 4.11. The gradient, which is shown in Equation 4.12, is used in the optimization routine. The termination tolerance for the optimization is set to \( 5^{-12} \). This value was chosen to ensure that the results will be very close to the analytical solution. The appropriate tolerance was found by setting the \( P \) matrix equal to identity and lowering the tolerance until the numerical optimization results matched the full feedback results.

This numerical optimization method is extremely computationally expensive. It
can take several hours to compute a solution for the flat plate mirror model. The size of the feedback matrix is equal to the number of actuators times the number of sensors, which means that it will grow rapidly as actuators and sensors are added to the model. For the flat plate mirror case with 42 actuators, Matlab runs out of memory during the optimization routine, and a solution cannot be found for this problem.

Now, both the full feedback and output feedback algorithms will be evaluated using the flat plate model with a simple bulk change in temperature.

4.2.3 Strain Gage Sensor Control on Flat Plate Mirror

Both the closed-form solution for the full feedback problem and the numerical optimization solution for the output feedback problem are evaluated on the flat plate mirror to determine the effectiveness of the control. This is the mirror model presented in Chapter 3, with geometric parameters given in Table 3.1. For this initial test, a 1°C bulk temperature change is used to test the control. As shown in Chapter 3, the bulk temperature change causes a distortion in the mirror because of the mismatch in CTE between the mirror material and the actuators.

Although the curvature within each mirror element is minimized by the control, the final wavefront error and the wavefront error correction factor in the mirror are used as performance metrics for the mirror surface. The correction factor is defined as the initial wavefront error divided by the corrected wavefront error in the clear aperture of the mirror where wavefront error is defined based on a Zernike decomposition of the surface displacements, as shown in Equation 2.4. For the flat plate mirror, the first 48 Zernike polynomials are used to represent the mirror distortions.

The results of both the full state and the output feedback problems are shown in Figure 4-3. Figure 4-3(a) shows the initial and final WFE in the mirror as the number of actuators is changed, and Figure 4-3(b) shows the WFE correction factor. As expected, the final surface error in the mirror is much smaller when full strain feedback is used. The WFE correction factor also improves as more actuators are added to the mirror. As shown in Figure 4-3(b), with full feedback, the WFE is corrected.
by more than two orders of magnitude, and even without using the full feedback, it is possible to improve the mirror surface by more than an order of magnitude. This is a completely linear model, so larger thermal disturbances could be applied, and the WFE correction factors would remain the same. Figure 4-4 shows the distorted and corrected shape of this mirror with 30 actuators.

Figure 4-4: 30 Actuator Mirror with Strain Gages: Uncorrected and Corrected Surface with 1°C Bulk Temperature Change

Figure 4-4(a) shows the initial distorted mirror surface and the corrected surface on the same axis. In the scale of this figure, the corrected surface appears to be perfectly flat. Figure 4-4(b) shows a zoomed in view of the corrected mirror surface. There are still some residual errors in the mirror surface, which can be seen in this figure. The center of the mirror has been flattened well, but there are still displacements around the edges of the mirror. This is not surprising since the control algorithm
does not attempt to control the mirror surface outside of the clear aperture. The peak displacements in the corrected surface are more than two orders of magnitude smaller than in the initial distorted mirror shape.

These results show that it is possible to correct the shape of the mirror surface using only embedded strain gages. However, the numerical optimization routine is very slow, and the problem does not directly minimize WFE. For these reasons, temperature sensors are also examined as a potential type of embedded sensor.

4.3 Control Using Temperature Sensors

For the control algorithm using embedded temperature sensors, the WFE is minimized directly. In this case, the WFE is described as a function of the actuator inputs and the disturbance:

\[
WFE = \sum u + \Gamma T \tag{4.13}
\]

\[
u = Fy \tag{4.14}
\]

where WFE is the wavefront error in the primary mirror described by the first 48 Zernike coefficients. Again, \(u\) is a vector of actuator commands and \(T\) is a vector of the thermal disturbances. In this problem, \(\Sigma\) is the actuator influence function matrix, and \(\Gamma\) is the disturbance influence function matrix where the influence functions relate inputs to the WFE of the mirror surface. The WFE is described by Zernike coefficients, so both influence matrices have 48 rows. The number of columns in the actuator influence matrix (\(\Sigma\)) corresponds to the number of actuators, while the number of columns in the disturbance influence matrix corresponds to the number of nodes in the model. The sensors, \(y\), are now a subset of the nodal temperatures described by the \(T\) vector.

\[
y = PT \tag{4.15}
\]
The $P$ matrix is used to select the nodal locations of the temperature sensors. The actuator commands, $u$ are a function of the sensors. Therefore, these equations can then be combined to create Equation 4.16, which describes the WFE in the mirror surface purely as a function of the thermal disturbances, $T$, the pointing matrix, $P$, and the feedback matrix, $F$.

$$WFE = \Sigma F P T + \Gamma T$$  \hspace{1cm} (4.16)

As before, the goal is to minimize $WFE^T WFE$, by finding the feedback matrix, $F$, which will correct the mirror to the best shape. The $WFE^T WFE$ control metric is calculated to be:

$$J = WFE^T WFE = T^T (\Gamma^T + P^T F^T \Sigma^T)(\Sigma F P + \Gamma) T$$  \hspace{1cm} (4.17)

$$J = T^T \Gamma^T \Sigma F P T + T^T \Gamma^T \Gamma T + T^T P^T F^T \Sigma^T \Sigma F P T + T^T P^T F^T \Sigma^T \Gamma T$$  \hspace{1cm} (4.18)

The minimum of this equation is then calculated by computing the derivative of $J$ with respect to $F$. This derivative is set to zero in order to compute the feedback matrix which results in the best final mirror correction.

$$\frac{\delta J}{\delta F} = \frac{\delta}{\delta F} (T^T \Gamma^T \Sigma F P T) + \frac{\delta}{\delta F} (T^T P^T F^T \Sigma^T \Sigma F P T) +$$

$$\frac{\delta}{\delta F} (T^T P^T F^T \Sigma^T \Gamma T) = 0$$  \hspace{1cm} (4.19)

By applying the rules of matrix differentiation, it is possible to solve for the feedback matrix, $F$:

$$2(\Sigma^T \Gamma TT^T P^T) + 2(\Sigma \Sigma^T F P T T^T P^T) = 0$$  \hspace{1cm} (4.20)

$$F = - (\Sigma \Sigma^T)^{-1} (\Sigma^T \Gamma TT^T P^T)(P T T^T P^T)^{-1}$$  \hspace{1cm} (4.21)
However, the term $\Sigma \Sigma^T$ is very poorly conditioned, making it difficult to perform the inversion. The condition number of $\Sigma$ is on the order of $10^8$. Therefore, when it is squared, it becomes $10^{16}$, which is the level of Matlab precision. In order to solve this problem, it is formulated as a least squares problem and the Matlab *lsqr.m* command is used to solve Equation 4.22 for $F$.

$$
(\Sigma \Sigma^T) F = -(\Sigma^T \Gamma \Sigma^T P^T)(P \Sigma \Sigma^T P^T)^{-1}
$$

Once the feedback matrix, $F$, has been computed, the actuator commands are calculated using Equation 4.14. These commands are applied to the actuators on a distorted mirror. The corrected shape of the mirror is then output from the finite element model.

### 4.3.1 Output Feedback Problem

A full feedback problem can be formed if all nodal temperatures are measured. This would be equivalent to using the identity matrix as the $P$ matrix in Equation 4.15 which selects the location of the sensors. Then the $T$ vector in Equation 4.22 will be known exactly. The difference in the $P$ matrix is the change between the full and output feedback problems using temperature sensors.

In reality, the number of sensors is limited, and it is not possible to have full feedback. The $P$ matrix is used to select which nodes have sensors. For the flat mirror, one temperature sensor is placed under each actuator; in addition, there is a sensor in the middle of each actuator hexagon in order to improve the temperature estimation, as shown in Figure 4-5. Multiple heights of temperature sensors are not used on the flat plate mirror model, because through-thickness gradients are ignored for a mirror of this thickness, as shown in Chapter 3. It is assumed that the temperature distribution on the optical surface of the mirror is the same as the distribution of the back surface. However, various thermal gradients across the mirror surface will be examined along with the simple case of the bulk temperature change.

Since the temperature gradient on the back surface of the mirror is unknown, an
attempt is made to minimize \((WFE)^T(WFE)\) for any temperature vector, \(T\). In order to do this, the cost function, \(J\), will be minimized independent of \(T\).

\[
J = T^T[(\Gamma^T + P^T F^T \Sigma^T)(\Sigma F P + \Gamma)]T
\]  
\begin{equation}
(4.23)
\end{equation}

The center part of this function is defined as \(A\):

\[
A = (\Gamma^T + P^T F^T \Sigma^T)(\Sigma F P + \Gamma)
\]  
\begin{equation}
(4.24)
\end{equation}

This variable, \(A\), is then substituted into the cost function, \(J\). Then, this equation is minimized by taking the derivative of \(J\) with respect to \(F\). The cost function should be at a minimum for any temperature disturbance when \(A\) is minimized.

\[
\frac{\delta J}{\delta F} = T^T \frac{\delta A}{\delta F} T = 0
\]  
\begin{equation}
(4.25)
\end{equation}

It is then possible to solve for the feedback matrix \(F\) which is not a function of temperature:

\[
F = -(\Sigma^T \Sigma)^{-1}(\Sigma^T \Gamma P^T)(PP^T)^{-1}
\]  
\begin{equation}
(4.26)
\end{equation}

This algorithm was evaluated on the flat plate mirror model, and only resulted in small corrections to the mirror surface. For the \(1^\circ C\) bulk temperature change on a 30 actuator mirror, the WFE correction factor was only 1.3. This method did not
work because it is necessary to multiple out the terms in Equation 4.23, and then take the derivative. This derivative will result in a term which includes the thermal disturbance. Therefore, the feedback control matrix, F, will depend on $T$, as shown in Equation 4.21.

Since it is not possible to control the mirror shape using one feedback matrix for any thermal gradient, it will be necessary to estimate the temperature distribution across the surface using the output of the sensors. The surface temperature is estimated by creating a matrix of possible temperature distributions and using the sensors to solve for the scaling factors.

$$T_{est} = \tau_1 T_1 + \tau_2 T_2 + \tau_3 T_3 \ldots = \tau T_{possible} \quad (4.27)$$

If the expected form of the thermal disturbances is known, then the vectors of the T matrix can be chosen to match the shape of these disturbances. For example, if it is known that the disturbance will be exponential in shape, a set of exponential functions can be used to form the $T_{possible}$ matrix. However, without knowledge of the disturbances, the Zernike polynomials can be used as basis functions. That is, $T_1$ will be a vector of constant value, $T_2$ and $T_3$ will be linear gradients in the x and y directions and so on. More columns can be added to the $T_{possible}$ matrix in order to accurately represent higher order thermal disturbances. If the actual disturbance has the form of one of these columns, then the estimation will work nearly perfectly.

### 4.3.2 Temperature Sensor Control on Flat Plate Mirror

This algorithm is now tested on the same simple model using a bulk temperature change of 1°C. This will cause the same initial WFE as seen in the results of Section 4.2.3. The results for the initial and final WFE and the WFE correction factor are shown in Figure 4-6.

When the temperature disturbance is a bulk temperature change, the surface error can be reduced to only a few nanometers by the shape control. This is possible because the $T_{possible}$ matrix contains a column which represents a bulk temperature change, so
the temperature distribution is estimated close to exactly. As the number of actuators is increased, the final WFE in the mirror decreases because the larger number of actuators can more accurately control the shape of the mirror. The WFE correction factor also increases with the number of actuators. This increase is nonlinear because the initial error is also increasing with the number of actuators used. The initial and final mirror shape for the 30 actuator mirror is shown in Figure 4-7.

Figure 4-7: 30 Actuator Mirror with Temperature Sensors: Uncorrected and Corrected Surface with 1°C Bulk Temperature Change

Figure 4-7(a) looks very similar to Figure 4-4(a), which showed the mirror correction using strain gage sensors. Again, the corrected mirror shape appears nearly flat on this scale. Figure 4-7(b) shows an enlarged view of the corrected mirror surface. This mirror has a slightly different shape than the mirror corrected using strain gage sensors, but the displacements are again located at the edges of the mirror. Again,
the peak displacements in the corrected mirror are more than two orders of magnitude smaller than in the initially distorted shape.

4.4 Conclusions

Control algorithms were developed for both types of embedded sensors. When the sensors are strain gages, numerical optimization must be used in order to find an analytical solution for the feedback matrix. When temperature sensors are used, the control law is much simpler. These control algorithms were tested for the simplest thermal disturbance case of a bulk temperature change. Both controllers were able to correct the mirror shape by more than an order of magnitude. Figure 4-8 shows a comparison of the correction factors when these two control laws are used.

![Comparison of Sensor Types: Bulk Temp Change](image)

(a) Initial and Final WFE

![Comparison of Sensor Types: Bulk Temp Change](image)

(b) WFE Correction Factor

Figure 4-8: Comparison of Strain Gages and Temp Sensors for Bulk Change

This figure shows that for the bulk temperature disturbance, the control algorithm based on temperature sensors is more effective than the strain gage control. This is true even when the full feedback strain gage control is used. These trends may change for more complex thermal disturbances which are not estimated perfectly using the temperature sensors. The next chapter will examine some of these disturbance patterns, as well as some potential variations in the mirror geometry.
Chapter 5

WFE Minimization on Simple Plate Model

The control algorithms developed in Chapter 4 are now applied to the flat plate model with errors caused by thermal disturbances. Both strain gages and temperature sensors are tested to evaluate the embedded sensing method. Multiple thermal disturbances, which were described in Chapter 3, are applied to the mirror. The effects of these disturbances on the mirror can then be characterized, and the performance of both controllers is analyzed for each disturbance. Mirror models with 12, 30 and 42 actuators are used to analyze each controller and determine the effect of adding more actuators.

Each algorithm is tested using both full feedback and a limited number of sensors in order to analyze the performance of the control. Some variation is then included in the model to find how much the controller performance degrades. Finally, the strain gages and temperature sensors are compared to determine which is more effective.

5.1 Strain Gage Control Results

The control algorithm using strain gages, which was developed in Section 4.2, is first analyzed using the flat plate mirror model. In Chapter 4, this control algorithm was used to correct a mirror which had been distorted by a bulk temperature change.
It was seen that the WFE in the mirror could be corrected by more than a factor of 100 when full feedback was used, and was corrected by more than a factor of 10 when a limited number of sensors was used. The bulk temperature change is the simplest disturbance form which could occur, and the distortions in this mirror are due entirely to the presence of the actuators. In this section, more complex thermal distortions will be used to analyze the controller performance.

5.1.1 Various Disturbance Types

The control law using strain gages is now tested for a variety of complex thermal disturbances. For this analysis, the number of strain gages used is equal to the number of actuators for each of the mirrors tested. The gages are located underneath the actuators on the top and bottom surface of the mirror. In reality, sensors would not be placed on the optical surface of the mirror. However, for modeling simplicity there is only one solid element through the thickness of the mirror, so it is not possible to measure the strain at intermediate locations through the mirror thickness.

The thermal disturbances which are used to test the control were shown in Figure 3-1. Each disturbance is analyzed for the 12, 30 and 42 actuator mirror in order to determine the effect of increasing the number of actuators. For each of these disturbances, the initial WFE in the mirror is calculated. Then the control algorithm is used to calculate the actuator inputs. These inputs are applied to the model as thermal loads on the piezoelectrics, and the final WFE of the corrected mirror is then computed. The WFE correction factor is calculated by dividing the initial WFE by the corrected WFE. These results for the linear, exponential and conical gradient are shown in Figure 5-1. The left column of plots show both the initial and final WFE in the mirror. The plots in the right column show the WFE correction factor.

The initial error for the linear and exponential gradients is approximately 100 nm. For the conical temperature gradient, the initial error is significantly higher, around 300 nm. By examining these figures, it can be seen that it is difficult to meet the 2 nm WFE requirement for these disturbances using strain gage sensors. In fact, when the number of sensors is limited and the output feedback form of the control law is used,
the final mirror shape will only meet the requirement for the exponential gradient. The difference in correction between the full feedback and output feedback results can be seen clearly in the right column of plots. Using full feedback significantly increases the correction factor in the mirror for all disturbance types. As expected, full feedback control performance is always better than output feedback because it uses a super-set of the output feedback sensors. In addition, the full feedback control law does not require numerical optimization, and so the solution is more exact.
Changing the number of actuators has an effect on both the initial and final WFE in the mirror. The initial error in the mirror is affected by the presence of actuators due to the mismatch in CTE between the actuator and mirror. For the linear temperature gradient, adding actuators to the mirror decreases the initial error in the mirror. Due to the lower CTE in the actuator elements, the mirror cannot deform as much in the regions with a negative temperature, as occurs in the linear gradient distribution. However, for both the exponential and conical temperature gradients, mirrors with more actuators have higher initial WFE errors. This is due to the same CTE mismatch effect that was seen for the bulk temperature change. Although adding actuators to the mirror can have the effect of inducing more initial error, it also allows higher order deformations to be controlled. Therefore, in most cases, the final WFE is reduced as actuators are added. By examining the WFE correction factors, shown in Figure 5-1(b), (d) and (f), it can be seen that the additional actuators are better able to reduce errors in the mirror shape.

Although the mirror did not achieve the desired 2 nm final WFE for most of these disturbances, it can be seen that the output feedback control results in correction factors of between 40 and 60 for all of the disturbance types tested, while using only 5-10% of the strain measurements available from the model. This means that the WFE in the mirror can be corrected by this amount using only strain sensing. Therefore, although the distortions due to a 1°C temperature disturbance could not be corrected to meet the requirements, this linear model could be used to determine how large a thermal disturbance could be corrected.

5.1.2 Parameter Variability

To this point, all of the controls have been tested on a mirror with nominal parameters. In reality, there will be some variability in the material parameters. For instance, each actuator will have a slightly off-nominal value of CTE and piezoelectric constant. Because the control law was designed for the nominal material parameters, it can be expected that the performance will degrade as the variability in parameters in increased.
In order to test this, the mirror model is generated with variation in the CTE of the actuator elements. This variation is a normal distribution around the nominal value. This analysis was performed on the 12 actuator mirror with a 1°C bulk temperature change. The actuators with off-nominal CTEs are created using a Matlab `randn` command. The controller is tested on the mirror with five different random CTE distributions, and the final WFE results are then averaged. The standard deviation of this normal distribution is gradually increased to determine how the performance is affected.

![Final WFE with Variation in Actuator CTE, Bulk Temperature Change](image1)

![WFE Correction Factor with Variation in Actuator CTE, Bulk Temperature Change](image2)

Figure 5-2: Effect of Variability in Actuator CTE, Control using Strain Gages on 12-Actuator Mirror with 1°C Bulk Temperature Change

Figure 5-2 shows how the final WFE and the correction factor change as the standard deviation of the actuator CTEs is increased. As would be expected, as the model deviates from the nominal parameters, the performance of the control degrades. This can be seen by the fact that the final WFE increases and the correction factor decreases, as the CTE variability increases.

This section showed that is possible to control the mirror shape for various thermal disturbances using only embedded strain gages. However, the numerical optimization routine used to solve the output feedback problem is computationally expensive. Therefore, the control using embedded temperature sensors will also be analyzed for these complex thermal distributions.
5.2 Results with Temperature Sensors

Now, the temperature sensor based control law which was formulated in Chapter 4 is analyzed using the flat plate model. It was previously shown that this control law provides large correction for a simple bulk temperature change in the mirror; now it is analyzed for performance with the more complex temperature distributions which were described in Chapter 3. To begin, the temperature distribution across the mirror surface will be estimated using the sensors as described in Section 4.3.1. The basis functions which are used to create the $T_{\text{possible}}$ matrix are the Zernike polynomials. Initially, three temperature vectors are used to form this matrix; using the Zernikes, these vectors will be a column of constants, a linear gradient in the x-direction and a linear gradient in the y-direction. Finally, improvements to this temperature estimation will be discussed.

5.2.1 Various Disturbance Types

It was shown in the previous chapter that the WFE due to a 1°C bulk disturbance could be corrected to below 2 nm for the 30 and 42 actuator mirrors using embedded temperature sensors. The next temperature disturbance which is analyzed is the across-surface linear gradient. The gradient applied has a temperature of 1°C at the highest point and decreases to a temperature of -1°C at the lowest point. As shown in Section 3.2.3, this will create a positive displacement of the optical surface on the hot side of the mirror and a negative displacement on the cold side of the mirror. The distorted and corrected mirror surfaces are shown in Figure 5-3.

The initial WFE for this disturbance is shown as a function of number of actuators in the left plot in Figure 5-4. It can be seen that the initial error due to this disturbance is above 100 nm. As was seen for the strain gage control, the mirror with 42 actuators has a smaller initial error than the mirror with 12 actuators. Again, adding actuators to the mirror improves the control performance. This can be seen by examining the plot on the right which shows the WFE correction factors for this disturbance. WFE correction factor is defined as the initial WFE divided by the final
Figure 5-3: Distorted and Corrected Mirror Shape for Linear Temperature Gradient WFE. The correction factors for the linear temperature gradient range from almost 200 for the 12 actuator mirror to 600 for the 42 actuator mirror. Each of the mirrors tested have a final WFE below 1 nm for all of the mirrors tested, meaning that linear gradients larger than 1°C to -1°C could still be controlled to below the WFE requirement of 2 nm.

![Figure 5-3](image.png)

(a) WFE, Linear Temp Gradient  
(b) WFE Correction, Linear Temp Gradient

Figure 5-4: WFE Correction Using Temperature Sensors

For this disturbance, the full feedback results are identical to the results when a limited number of sensors are used. It can be seen in Figure 5-4(a) that the full feedback and output feedback results are the same. The error between the actual surface temperature and the estimated temperature is nearly zero, which means that the limited number of sensors performs as well as if a sensor is used at every node.
This occurs because the across surface gradient is one of the vectors used to form the $T_{\text{possible}}$ matrix for temperature estimation.

The exponential and conical shaped disturbances are now analyzed using the same mirror model. The results with these disturbances are shown in Figure 5-5. For these disturbances, there is a large difference in the correction achieved when a limited number of sensors is used as compared to the full feedback results. This occurs because the $T_{\text{possible}}$ matrix only includes three possible temperature distributions, which is not enough to correctly estimate the temperature distribution. Using full temperature feedback, the mirror can be corrected to below the 2 nm WFE requirement for both the exponential and conical temperature distributions. However, when the number of sensors is limited, the mirror with the conical gradient can only be corrected to approximately 10 nm, even when the number of actuators is increased.

![WFE and Correction Factor for Exponential and Conical Gradients](image)

(c) WFE, Conical Temp Gradient  (d) WFE Correction, Conical Temp Gradient

Figure 5-5: WFE and Correction Factor for Exponential and Conical Gradients

The plots in the right column of Figure 5-5 show the WFE correction factors for these disturbance types. It can be seen that the correction factors are an order
of magnitude higher for the full feedback case. This shows that the temperature estimation will need to be improved by adding more basis functions to the $T_{\text{possible}}$ matrix.

5.2.2 Improvement of Temperature Estimation

The previous section showed that using three possible temperature distributions to form the $T_{\text{possible}}$ matrix does not provide a good estimate of the actual temperature for the circular or exponential gradients. More Zernike polynomials are added to this matrix in order to improve the estimation. Figure 5-6 shows the actual and estimated temperature distributions for the exponential temperature gradient using three and six Zernike functions. The estimated surface based on only three Zernike polynomials, shown in Figure 5-6(a), is linear, and therefore does not match the actual distribution well. When six Zernike polynomials are used to estimate the temperature distribution, as in Figure 5-6(b), the combination of linear and astigmatism terms causes the estimated temperature to match the actual temperature nearly exactly.

![Figure 5-6: Estimation of Exponential Temperature Distribution](image)

(a) Three Zernike Polynomials  
(b) Six Zernike Polynomials

Although Figure 5-6 only shows improvement as more Zernike terms are used in the estimation, for each actual temperature distribution, there is an optimal number of possible temperatures which should be modeled. This is because as too many columns are added to the matrix of possible temperatures, it is no longer possible
to estimate the temperature correctly. Figure 5-7 shows the change in the final WFE for the 30 actuator mirror when the number of basis functions in the $T_{\text{possible}}$ matrix is varied. It can be seen that adding columns to this matrix will improve the performance when there is an exponential gradient. As more functions are added, the final WFE approaches the result which is obtained when full feedback is used. For the conical temperature distribution, adding basis functions initially improves the controller performance. However, the performance begins to degrade once more than eight Zernikes are included in the estimation. For the conical disturbance, the eight Zernike basis functions were only able to improve the correction factors slightly when compared to using three basis functions. There is still approximately a factor of ten difference between the output feedback and full feedback performance. This shows that even when many functions are used, the Zernike polynomials are not able to capture the shape peak in the conical temperature distribution very well.

![Temperature Estimation Improvement - Exponential Temp Gradient](image)

![Temperature Estimation Improvement - Conical Temp Gradient](image)

Figure 5-7: Number of Basis Functions versus Performance of Temperature Sensor Control

Based on the results shown in Figure 5-7, the number of basis functions used to estimate the temperature distribution is increased to eight. This means that Zernike functions representing piston, tip, tilt, focus, astigmatism, coma and spherical distributions are used as possible temperature distributions. The control algorithm is tested again for the conical and exponential gradients, and the results are compared to the earlier results when only three basis functions were used. These results are shown in Figure 5-8. The plot on the left shows the control results with an exponential temperature disturbance. For this disturbance, adding the five additional
Zernike forms significantly improves the control performance. The correction factor is increased by nearly an order of magnitude. The plot on the right shows the control results with a conical shaped temperature disturbance. Using eight basis functions provides only a slight improvement in the control performance over using three basis functions. There is still a large difference between the full feedback and output feedback results. Therefore, the Zernike polynomials do not do a good job at estimating the conical shaped temperature disturbance but they are able to estimate the exponential gradient very well.

Figure 5-8: Controller Performance with Improved Temperature Estimation

As more knowledge is gained about the expected form of the thermal disturbances, it might be possible to create a more accurate set of columns for the $T_{\text{possible}}$ matrix. At this time, it is assumed that the disturbance is unknown. For future results using the temperature sensors, eight Zernike polynomials are used as the basis functions for temperature estimation.

5.2.3 Parameter Variability

Section 5.1.2 showed the effect of variation in material parameters when the strain gage control algorithm was used. Now, the temperature sensor control is analyzed with variation in the CTE of the actuators. Figure 5-9 shows the final WFE and the WFE correction factor for a bulk temperature change in the mirror as the variation in the actuator CTE is increased. Again, it is assumed that the CTE has a normal distribution centered around the nominal value; the standard deviation of this distri-
bution is increased and the effect on the control is determined. As would be expected, it can be seen that the final WFEs increase, and the correction factors decrease as the standard deviation of the actuator CTEs is increased.

![Diagram](image)

(a) Final WFE with Parameter Variability  
(b) WFE Correction with Parameter Variability

Figure 5-9: Effect of Variability in Actuator CTE, Control using Temp Sensors

When there is no variation in the value of the actuator CTE, the mirror can be corrected to 2.9 nm with 12 actuators and 1.5 nm with 30 actuators. Adding even a 5% standard deviation to the actuator CTE increases the final WFE in the mirror to 18 nm and 20 nm for the 12 and 30 actuator mirrors, respectively. Although the temperature sensors have a much better performance than the strain gages when there is no variability in the material parameters, adding and variability into the model quickly reduces the control performance to the same level as the stain gage control. When Figure 5-9(a) is compared to Figure 5-2(a), it can be seen that the performance of the temperature sensors and the strain gages is very similar for the 12 actuator mirror with variability in the actuator CTE.

It was shown in this section that embedded temperature sensors could be used to control the mirror shape for a number of thermal distributions. The control algorithm using temperature sensors is much simpler to implement and runs much faster. However, the temperature sensor control is very sensitive to variability in the actuator CTE, which could cause problems when this algorithm is implemented on a real system.
### 5.3 Conclusions from Simple Model

The performance of the strain gages and temperature sensors can now be compared in order to determine which method is better. The two types of sensors are compared for the linear, exponential and conical temperature gradients. For each of these disturbances, both the full and output feedback results are shown. In this model of the mirror, sensor noise is not considered.

Figure 5-10 shows these results for the linear temperature gradient. The initial error, shown in green, is approximately 100 nm. Results are shown for both strain gages (using both full feedback and output feedback) and temperature sensors. Only one set of results is shown for temperature sensors, because the output feedback results exactly match the full feedback results for the linear temperature distribution, as seen in Section 5.2. In this figure, the 2 nm requirement is shown by the dashed line. It can be seen that the temperature sensor control performs much better than the strain gages in this case. The WFE requirement can be met using the 12, 30 or 42 actuator mirror when temperature sensors are used. However, 42 actuators were necessary to meet the WFE requirement when using full feedback control from strain sensors.

![Comparison of Sensor Types: Linear Temp Change](image)

**Figure 5-10: Comparison of Sensors for Linear Temperature Gradient**

Next, an exponential temperature gradient was applied to the mirror. The results with this disturbance are shown in Figure 5-11. Again the initial error in the mirror is around 100 nm. However, for this type of disturbance, the correction factors are much higher, and it is possible to reduce the error in the mirror to below the WFE
requirement using either temperature sensors or strain gages. For the exponential
distribution, the full feedback and output feedback results using temperature sensors
are different. These results used eight Zernike basis functions, which improves the
performance of the temperature sensor control, as shown in Section 5.2.2. For this
disturbance type, the control using a limited number of temperature sensors actually
performs better than the control which using strain feedback from every element in
the model (full feedback).

Figure 5-11: Comparison of Sensors for Exponential Temperature Gradient

The final disturbance type analyzed is the conical distribution. The initial errors
for this disturbance type are significantly higher than for the linear or exponential
disturbances. The final errors are also generally higher than were seen previously.
The results for each sensor type are shown in Figure 5-12. As seen in Section 5.2.2,
the eight Zernike basis functions are not able to improve the temperature estimation
enough to meet the WFE requirement. It can be seen in Figure 5-12 that there is
still a large difference in the performance of the temperature sensor control using
full feedback as compared to the performance with a limited number of sensors. In
order to improve this performance, it will be necessary to improve the temperature
estimation, which could be done by changing the set of basis functions used.

In general, the control algorithm based on embedded temperature sensors results
in better performance than control based on strain gages. Although the temperature
ensors were not able to control the mirror well for a conical disturbance, they per-
formed better for all other distributions. In addition, it is possible to improve the
Figure 5-12: Comparison of Sensors for Conical Temperature Gradient

temperature estimation by using more sensors. As more information about the shape of the expected disturbances is known, the temperature estimation can be improved until the output feedback results approach the results achieved when full feedback is used. It can be seen that the full feedback results using temperature sensors are superior to the full feedback results using strain gages for all of the temperature distributions analyzed. The temperature sensors are also favorable because the control algorithm can be written to directly minimize the WFE in the mirror, instead of minimizing the curvature.

When some variability was added to the actuator CTE in the model, it was seen that the performance of the temperature sensor control degraded more than the performance of the strain gage control. Therefore, for a real system, the performance with these two types of sensors might be very similar. However, the temperature sensor control algorithm does not require a numerical optimization routine, so it can be solved much more quickly. As the size of the model increases, as it will for the rib-stiffened mirrors, this becomes very important.
Chapter 6

WFE Minimization on a Rib-Stiffened Mirror

It was shown in Chapter 5 that the embedded sensors could be used to control the shape of a simplified structure which is representative of a mirror. This structure was a flat plate with actuators for shape control that were offset from the back surface of the mirror on posts. However, many lightweight mirrors are now designed using the rib-stiffening concept shown in Chapter 2. In this type of mirror, surface-parallel actuation is created by embedding the actuators in the rib structure. In this chapter, the control algorithm using embedded temperature sensors is applied to a rib-stiffened mirror.

Embedded temperature sensors, and not strain gages, are used to control the rib-stiffened mirror. The previous chapter showed that temperature sensors are able to correct the mirror better than strain gages for most thermal disturbances. In addition, the control algorithm using temperature sensors could be written directly for WFE minimization, and it was not necessary to use a numerical optimization routine. The numerical optimization routine was very slow and could not be run on larger models due to computer memory constraints. The rib-stiffened mirror contains more actuators and nodes than the simplified model, and so the numerical optimization would be a problem.

The presence of the rib structure will cause the deformations of these mirrors to
have much higher spatial frequency content than was seen in the plate structures. For this reason, the controller for the rib-stiffened mirror will minimize the WFE in the mirror as described by 156 Zernike polynomials, rather than the 48 polynomials used previously. The influence functions of the actuators and disturbances will therefore be composed of 156 Zernike coefficients. This was not necessary in the case of the flat plate mirror, because the deformations were described well by 48 Zernike terms. In addition, for the rib-stiffened mirror, the initial and final WFE will be computed based on a root sum square of all of the points on the optical surface of the mirror, as shown in Equation 6.1. This ensures that no residual high order deformations are neglected when the final WFE is calculated. This also provides a truth measure that is distinct from the control objective and feedback sensor outputs.

\[
WFE = \sqrt{\sum \frac{z^2}{n}}
\]  

(6.1)

In this chapter, the control will be applied to a single hexagonal segment of a mirror. This is a smaller model than the entire six segment mirror, shown in Figure 2-6. For a hexagonal segment, actuators can be placed to the edges of the mirror, so it is not necessary to neglect the edges when the wavefront is measured. The control algorithm will be analyzed for the same thermal disturbances which were described in Chapter 3. In addition, the effect of changing mirror parameters will be tested using the parametric model of the rib-stiffened mirror.

### 6.1 WFE Minimization Results

To begin, the control law will be tested for a number of temperature distributions as the number of ribs on the mirror is changed. Figure 6-1 shows mirrors with two, three and four rib rings which are used to test the control. An actuator is embedded in the center of each rib, so the number of actuators is directly determined by the number of ribs. For each of these mirrors, it is important to ensure that there are enough grid points on the optical surface to capture all 156 Zernike shapes which will be minimized. If there are too few grid points, then aliasing will occur and the higher
order polynomials will look the same as lower order shapes. To be conservative, the number of grid points in the model will be at least three times the number of Zernikes. This means that for the one meter hexagonal segment used, a minimum mesh density of 24 elements per meter must be used.

![Rib-Stiffened Hex Segment with Embedded Actuators (Shown in Red)](image)

Figure 6-1: Rib-Stiffened Hex Segment with Embedded Actuators (Shown in Red)

To begin the analysis of the controller on the rib-stiffened mirror, all properties of the mirror will be held constant except for the number of ribs. The nominal geometric parameters for the rib-stiffened mirror are shown in Table 6.1. The areal density of the mirror is kept at a constant value of 15 kg/m$^2$. As discussed in Chapter 2, this means that the size of the ribs will be decreased as the number of ribs is increased.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>1 m</td>
</tr>
<tr>
<td>F #</td>
<td>1</td>
</tr>
<tr>
<td>Areal Density</td>
<td>15 kg/m$^2$</td>
</tr>
<tr>
<td>Rib Aspect Ratio</td>
<td>4</td>
</tr>
<tr>
<td>Facesheet Mass Ratio</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 6.1: Hex Segment Properties

The same controller described in Chapter 4, using one temperature sensor for each actuator, is used to control the rib-stiffened hex segment. The four thermal disturbances which were used in Chapter 5 are again applied to the mirror. These disturbances are a 1°C bulk temperature change, a linear temperature gradient from 1°C to -1°C across the diameter of the mirror surface, an exponential temperature gradient from 1°C to 0°C across the surface, and a conical shaped gradient with a peak temperature of 1°C. These temperature distributions are shown in Figure 3-1.
The shape control is then applied to the rib-stiffened hexagonal segment with the parameters shown in Table 6.1. The results of the control for each of the four temperature distributions are shown in Figure 6-2. It can be seen in each of these plots that the initial error due to the thermal disturbance is significantly higher than for the flat plate model. For the bulk temperature change on the flat plate, the initial error was between 600 and 1000 nm. When this same temperature change is applied to the rib-stiffened hexagon, the initial error is 3000 nm for the mirror with 42 actuators and 6500 nm for the mirror with 156 actuators.

The flat plate mirror had a diameter of only 40cm, while the hex segment has a 1-meter diameter. Also, the hex segment is curved, which will cause it to deform under a bulk temperature change. Finally, the rib-stiffening structure causes a non-uniform stiffness in this mirror. Therefore, even without the embedded actuators, a bulk temperature change will cause the hex segment to deform in the focus shape seen earlier. In each of these plots it can be seen that the initial error increases as the number of actuators, and therefore ribs, is increased. Therefore, although
more actuators can be beneficial in controlling deformations, the rib structure itself contributes to the errors when a thermal disturbance is present.

Figure 6-2 also shows that the final WFE in the mirror is reduced when more actuators are present. Even though the initial errors are increasing, the final WFE in the mirror decreases for all of the disturbance types as the number of ribs increases. It can be seen in Figures 6-2(c) and 6-2(d) that the full feedback control works much better than the control using a limited number of sensors.

6.2 Parameter Variations

The mirror geometry can be easily changed using the parametric model for the rib-stiffened mirror which was described in Chapter 2. In Chapter 2, the parametric model was used to determine the best geometry for a rib-stiffened mirror for dynamic performance. In this section, the same model will be used to analyze the effects of changing mirror geometry for shape control of thermal deformations. Each time a geometric parameter is altered, it is necessary to re-generate the influence functions, which is a slow process. Therefore, analyzing many different mirror geometries requires more computation time for the shape control problem than for the dynamic performance analysis which was shown in Chapter 2.

6.2.1 Areal Density Variation

Reducing the areal density of a mirror is a major design goal for future space telescopes. Therefore, the parametric mirror model was designed with areal density as an input. This parametric model can then be used to determine the effect on the control of reducing the areal density. As in Chapter 2, areal densities in the range of 5kg/m$^2$ to 15kg/m$^2$ will be studied. Changing the areal density of the mirror will affect both the initial error in the mirror and the correction factors. Figure 6-3 shows the initial WFE in the rib-stiffened mirror as the areal density is changed. For all disturbance types, the errors decrease as the areal density is increased. This is expected given that the stiffness of the mirror increases with areal density, as seen in Chapter 2.
The WFE correction factor will be affected by the change in areal density because the influence of each actuator changes with the mirror stiffness. Figure 6-4 shows the influence function of actuator #1 for the 90 actuator rib-stiffened mirror as the areal density of the mirror is increased from 5 \( \text{kg/m}^2 \) to 15 \( \text{kg/m}^2 \). The color in these figures represents the z-displacements of each node in the mirror model. The color scales are identical in each of these figures. It can be seen that the actuator has a larger and more distributed effect on the lower density mirror. As the areal density is increased, the maximum displacement is reduced, and the region affected by the actuator becomes smaller.

The full matrix of influence functions is generated for a 90-actuator mirror using areal densities of 5, 10 and 15 \( \text{kg/m}^2 \). All of the other geometric parameters are kept at the nominal values shown in Table 6.1. The control algorithm is then applied to each of these mirrors. The initial error under each of the thermal disturbances was
shown in Figure 6-3. Figure 6-5(a) shows the final errors in the corrected mirror for each disturbance type, and Figure 6-5(b) shows the WFE correction factors.

For both the bulk and linear temperature gradients, the final WFE in the mirror is smaller for the lower areal density mirrors. For the exponential and conical gradients, the final WFE is slightly smaller for the high areal density mirrors. In order to determine why the different temperature distributions show opposite trends for the final WFE, both the initial and final mirror surfaces will be examined more closely.

The initial deformed shape of the mirror segment under a bulk temperature change and an exponential temperature gradient is shown in Figure 6-6. It can be seen that both of these deformations are composed of a low spatial frequencies. The initially deformed shape due to the bulk temperature change is a symmetric shape similar to the focus mode. The exponential temperature gradient creates a more complex, non-symmetric shape.

Figure 6-7 shows the residual surface displacement after the control is applied in the mirror segment, for a bulk temperature change. The initial errors due to the bulk temperature change are due to the presence of the rib structure and actuators. It can be seen that the residual deformations are high order and match the patterns of the rib/actuator structure. These residual errors are local curvatures of the rib elements due to the actuator. The mirror facesheet has been flattened in the cells; however the rib elements curved in order to flatten these facesheet elements. The errors due to the local curvature are larger in a heavier mirror because the influence of each actuator is more localized. For the lighter mirrors, the effect of each actuator is spread across
the mirror surface, so residual errors are smaller.

It was seen in Figure 6-5(a) that while the final errors in a mirror with a bulk temperature change are reduced as the areal density is decreased, the final errors with an exponential or conical temperature gradient increase with lower areal density. In order to understand why these two temperature distributions show opposite trends, the residual errors in the mirror surface with an exponential gradient are shown in Figure 6-8. This surface error is of a much lower spatial frequency than the residual errors seen for the bulk temperature change. The residual error in this mirror is not simply due to local curvature near the actuators, instead it is error due to the initial disturbance which could not be corrected by the 90 actuator mirror. The final error is therefore lower in the 15 kg/m² mirror because the initial error was lower.

The correction factors, defined as initial WFE divided by final WFE, are shown in Figure 6-5(b). This figure shows that the correction factors are much higher for low
areal density mirrors for the bulk and linear distributions. This is the case because the initial error is so much higher, and the surface can be corrected to a smaller final error because the influence of each actuator is more distributed. The correction factors for the exponential and conical gradients remain nearly constant as the areal density is changed.

### 6.2.2 Rib Aspect Ratio Variation

In Chapter 2, it was shown that the aspect ratio of the rib cross section has a significant impact on the stiffness of the mirror. This parameter can now be changed to determine its effect on the mirror shape control. The rib aspect ratio was previously defined at the height of the rib cross section divided by the width. The initial errors in the same 90-actuator mirror with an areal density of 15 kg/m$^2$ are shown in Figure 6-9. As the rib aspect ratio is increased the mirror becomes stiffer, and the initial errors decrease. This is the same effect seen in the previous section when the areal density of the mirror was increased.

As shown for the changing areal density, the increase in mirror stiffness with increasing rib aspect ratio will cause the influence of each actuator to become more localized. The effect on the final WFE in the mirror is therefore similar to the effects seen for changing areal density. The final WFE in the mirror is shown for each temperature distribution in Figure 6-10. Again, the final WFE in the mirror increases as the mirror becomes stiffer for the bulk and linear temperature changes. The final WFE decreases with higher rib aspect ratios for the exponential and conical
Finally, the WFE correction factor in the mirror surface can be examined for each of the thermal disturbances. These correction factors are shown in Figure 6-11. The correction factors, for every type of temperature disturbance, decrease as the rib aspect ratio is increased. It was seen that the final WFE is reduced as aspect ratio increases for the exponential and conical temperature gradients. However, the WFE correction factors are higher for mirrors with low rib aspect ratios for all of the temperature distributions because the initial errors are much higher in mirrors with low aspect ratio ribs.
Figure 6-11: WFE Correction Factor for Rib-Stiffened Mirror, Variation in Rib Aspect Ratio

6.3 Rib-Stiffened Mirror Control Conclusions

The control algorithm presented in previous chapters was analyzed on the rib-stiffened mirror. Initially, three different mirrors were used: two rings of rib-stiffening with 42 actuators, three rings of rib-stiffening with 90 actuators, and four rings of rib-stiffening with 156 actuators. The results of this control show that it is possible to use embedded temperature sensors to control the shape of the mirror. For the temperature disturbances tested, it was not possible to reach the 2 nm WFE requirement. However, high correction factors were achieved for all disturbance types, especially when a larger number of actuators were used. Again, the results were not as good for the exponential and conical disturbances due to the errors in estimating the temperature.

The parametric model of the lightweight mirror was then used in order to determine the effect of varying geometry on the shape control. Changing the mirror geometry will cause the actuator influence functions to change. The effect of an actuator on a stiffer mirror will be more localized than for a more flexible mirror. Both the areal density of the mirror and the aspect ratio of the ribs were varied. It could be seen that heavier mirrors with high aspect ratio ribs will have a much lower initial WFE for any of the temperature distributions analyzed. These are stiff mirrors, and so it is not surprising that the initial error is lower.
For the bulk and linear temperature changes, the final WFE is lower when the mirror is less stiff. Therefore low areal density and low rib aspect ratio cause the final errors to be lower. This is because the actuator influence functions are more distributed for flexible mirrors, so local residual curvature is smaller. For the exponential and conical temperature change, the WFE correction factor remains nearly constant with changes in both areal density and rib aspect ratio. The residual surface error in the mirror with these disturbance types is not due to local curvature near actuators but is error that could not be corrected with the given number of actuators. Therefore the best geometry mirror for shape control of thermal effects depends on the type of temperature disturbance expected.
Chapter 7

Conclusions

7.1 Thesis Summary

This thesis analyzed the design of a lightweight mirror for space telescope applications. This type of analysis has become more necessary due to the increasing aperture sizes proposed for the next generation of space telescopes. In this thesis, the dynamic performance of a lightweight mirror is analyzed with many different geometry variations. In addition, a shape control algorithm using embedded sensing is proposed in order to reduce errors in the primary mirror as a result of thermal disturbances. This algorithm is then used to control the rib-stiffened mirror, and the control is analyzed as a function of changing mirror geometry.

To begin, the dynamic performance of an un-controlled mirror was analyzed. Chapter 2 presented the MOST tool, which is used to generate an integrated model to analyze the performance of various space telescope architectures subject to on-orbit dynamic disturbances. In order to analyze these different telescope architectures, a parametric model of the entire system was created. Because a goal for the next generation of space telescopes is to significantly reduce the areal density of the primary mirror, a parametric model of a rib-stiffened primary mirror was created. The desired mirror areal density is used as an input to this parametric model, so that geometry variations could be performed under a constant areal density. This allows mirror designs of equal mass to be compared. Chapter 2 describes the development
of this mirror model, which was then used to analyze the homogeneous dynamics of the mirror. Various geometric parameters were altered in order to determine an optimal geometry for the mirror design. It was determined that for a constant areal density rib-stiffened mirror, using fewer, large ribs resulted in a stiffer mirror than if many small ribs were used. It was also found that the mirror stiffness was increased by using tall, thin ribs and putting most of the mirror mass into the rib structure. The full dynamic disturbance to performance analysis showed that in general, the heavier, monolithic mirror designs result in better optical performance. Therefore, if lightweight and segmented mirrors are desired due to cost concerns and launch constraints, it is necessary to use active control of the mirror.

Next, a method for mirror shape control in the presence of thermal disturbances was presented. In Chapter 3, the thermal disturbance models which were applied to the mirror were presented. In order to gain an understanding of the effect of these thermal disturbances, a simple one dimensional beam structure was used. Using this simple model, the effects of a bulk temperature change, a linear temperature gradient across the beam, and a through-thickness temperature gradient were predicted. In addition, it was determined that through-thickness temperature gradients could be ignored for a thin silicon carbide mirror due to the high conductivity of the material. The shape control algorithms were first tested using a flat plate mirror model, which is a simplified version of the rib-stiffened mirror shown previously.

The traditional method of shape control would use a wavefront sensor located away from the primary mirror in the optical path. However, most wavefront sensors require a beam splitter which divides the beam of light into an amount for sensing and the remainder for image formation. In addition, errors could occur due to the separation of the primary mirror and the wavefront sensor. This thesis presented a shape control algorithm which uses only sensors embedded in the primary mirror in order to avoid these problems. Shape control algorithms were developed for both embedded strain gages and embedded temperature sensors. The required resolution for each of these sensor types was discussed in Chapter 3, and it was determined that either type of sensor could be used to correct the surface to a required error of $\frac{\lambda}{20}$. 

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although temporal averaging might be required. In Chapter 4, both full feedback and output feedback algorithms were developed. For the strain gage sensors, it was necessary to use a numerical optimization routine to solve for the feedback matrix. For the temperature sensors, the form of the thermal disturbance was estimated using feedback from a limited number of sensors. Chapter 5 showed the results of these two control algorithms on the flat plate mirror model with a varying number of actuators. The control algorithms were analyzed for each of the thermal disturbances presented in Chapter 3.

Once it was determined that shape control with embedded sensing worked on the flat plate model, the control algorithm using embedded temperature sensors was applied to the rib-stiffened mirror which was presented in Chapter 2. In Chapter 6, the parametric model of a rib-stiffened mirror was used to analyze the surface correction using this control algorithm. The initial errors due to the thermal disturbances were significantly higher for a lightweight rib-stiffened mirror than for the flat plate mirror examined in Chapter 5. Due to these high initial errors, it was not possible to achieve the 2 nm wavefront error requirement. However, it was possible to achieve more than an order of magnitude correction in the WFE. When the number of actuators used was increased to 156, it was possible to achieve more than two orders of magnitude correction for a bulk temperature change. Therefore, this algorithm would be useful as long as the thermal disturbances are smaller than the 1°C used in this analysis.

The parametric model for the rib-stiffened mirror was then used to test the shape control results. The mirror control was tested using two, three and four rings of rib-stiffeners. The initial errors in the mirror were lower when fewer ribs are used. This corresponds to the fact that fewer ribs will create a stiffer mirror, as shown in Chapter 2. However, because the number of actuators is increased along with the number of ribs, a lower absolute final error could be achieved by using more ribs. Additional mirror parameters such as the areal density and the rib aspect ratio were also varied to determine their effect on control. Initial errors due to all of the thermal disturbances were lower for the stiffer mirror designs. However, the actuator influence functions were more localized for these mirrors. The more distributed actuator
influence which exists in more flexible mirrors causes the final error to be lower for some thermal disturbances. Therefore, the best mirror design for performance with thermal disturbances is dependent on the types of disturbances expected.

The parametric mirror model developed in this thesis was useful in analyzing mirror design for both dynamic performance and shape control. The mirrors which were shown to have higher stiffness in the early analysis had better optical performance in the dynamic analysis and smaller errors due to thermal disturbances. However, when active controls were added to this model, more flexible mirrors, with more embedded actuators, could be corrected to smaller final errors. Therefore, the most favorable geometry for a lightweight mirror will depend on the expected disturbances.

7.2 Contributions

This thesis made the following contributions to the area of lightweight mirror design and control:

- A parametric model of a lightweight, rib-stiffened mirror was developed. This model used the desired areal density of the mirror as an input so that the effects of changing the mirror geometry, under constant mass, could be determined.

- Using this parametric model, it was determined that, for a constant areal density mirror, using a smaller number of large ribs will result in a stiffer mirror that if a large number of small ribs are used.

- Static shape control algorithms were developed to reduce mirror errors due to thermal disturbances using only embedded sensing.

- The shape control algorithm was applied to the parametric rib-stiffened mirror. It was determined that although stiffer mirror designs will have a smaller initial error under a given thermal disturbance, more flexible mirrors can be corrected to a smaller final error due to the broader influence of each actuator.
7.3 Recommendations for Future Work

The integrated model described in Chapter 2 was developed to analyze telescope performance with disturbances due to on-orbit dynamics. The model developed is completely linear and does not include effects such as buckling. It is particularly important to analyze buckling effects because it was determined in Chapter 2 that optimal mirror designs will utilize thin ribs and a thin mirror facesheet. It is possible that these thin structures will begin to buckle. This thin facesheet could also lead to print-through effects where residual stressed during manufacturing and polishing cause the mirror to distort in a shape that corresponds to the rib pattern.

In the future, this model could be extended to include the effects of launch load disturbances. The potential effect of launch loads on the mirror design has not yet been examined, and will likely drive the mirror design to higher stiffness. It is possible that the actuators embedded in the mirror surface could be used to alleviate stresses in the mirror during launch. In order to analyze the full telescope during launch, it will be necessary to model the stowed configuration of the mirror.

The shape control algorithm presented assumed a perfect model of the mirror, with all sensors and actuators working as expected. This control problem should be analyzed with sensor noise included in the model. In addition, the actuators were given no operational limits. In a real system, there are voltage limits on the actuators. It is also possible that an actuator could perform less than expected or even fail completely. In addition, the current shape control algorithm uses a very simple method to perform the temperature estimation. With additional knowledge of expected thermal disturbances, this estimation could be improved. A heat transfer model could be linked to the structural model in order to analyze thermal distributions in the mirror. This would provide better information about expected temperature distributions, and would improve the estimation.

The current shape control algorithm has many limitations. To account for some of these, the control law developed in Chapter 4 would be rewritten as a slow dynamic problem. This dynamic problem could be written in state-space form, which would
allow optimal control strategies to be used to minimize the effects of thermal disturbances. This would also allow the dynamic and thermal models to be combined, so that the active control can be analyzed for multiple disturbance sources.

The results shown in this thesis indicate that it would be possible to control the shape of a primary mirror using only embedded sensors. Future work on this topic should include an experimental demonstration of this concept. Either a simple surface-parallel actuated plate or a complete rib-stiffened mirror could be used to validate the shape control results shown in this thesis.
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